18. $f''(x) = \cos x + e^{3x}$ $\quad f(0) = \frac{1}{2}$ $\quad f'(0) = \frac{1}{3}$

$$f'(x) = \int f''(x)$$

$$= \int (\cos x + e^{3x})\,dx$$

$$= \int \cos x\,dx + \int e^{3x}\,dx$$

$u = 3x \quad du = 3\,dx$

$$= \sin x + \frac{1}{3}\int 3e^{3x}\,dx$$

$$= \sin x + \frac{1}{3}\int e^{u}\,du$$

$$= \sin x + \frac{1}{3}e^{u} + C$$

$$f'(x) = \sin x + \frac{1}{3}e^{3x} + C$$

$f'(0) = \frac{1}{3}$ → $x = 0$, $f'(x) = \frac{1}{3}$

$$\frac{1}{3} = \sin 0 + \frac{1}{3}e^{3(0)} + C$$

$$\frac{1}{3} = \frac{1}{3} + C$$

$$0 = C$$

$$f'(x) = \sin x + \frac{1}{3}e^{3x} + 0$$

$$f(x) = \int f'(x)$$

$$= \int \left(\sin x + \frac{1}{3}e^{3x}\right)dx$$

$$= \int \sin x\,dx + \frac{1}{3}\int e^{3x}\,dx$$

$$f(x) = -\cos x + \frac{1}{3}\cdot\frac{1}{3}e^{3x} + C$$

$$f(x) = -\cos x + \frac{1}{9}e^{3x} + C$$

$f(0) = \frac{1}{2}$ → $x = 0$, $f(x) = \frac{1}{2}$

$$\frac{1}{2} = -\cos 0 + \frac{1}{9}e^{3(0)} + C$$

$$\frac{1}{2} = -1 + \frac{1}{9}(1) + C$$

$$\frac{1}{2} + 1 - \frac{1}{9} = C$$

$$\frac{3}{2} - \frac{1}{9} = C$$

$$\frac{27}{18} - \frac{2}{18} = C$$

$$\frac{25}{18} = C$$

So

$$f(x) = -\cos x + \frac{1}{9}e^{3x} + \frac{25}{18}$$