Rolle's Theorem and the Mean Value Theorem

Rolle's Theorem

Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If

f(a) = f(b)

Then there is at least one number c in (a, b) such that f'(c) = 0

1. Explain why Rolle's Theorem does not apply to the function even though there exist a and b such that f(a) = f(b)
(similar to p.216 #1-4)

$$f(x) = \frac{1}{|x-2|}$$

$$[-4,8]$$

2. Find the two x-intercepts of the function f and show that f'(x) = 0 at some point between the two x-intercepts)
(similar to p.216 #5-8)

$$f(x) = x^2 + 2x - 15$$

3. Find the two x-intercepts of the function f and show that f'(x) = 0 at some point between the two x-intercepts)
(similar to p.216 #5-8)

$$f(x) = 4x\sqrt{x+2}$$

4. Determine whether Rolle's Theorem can be applied to f on the closed interval [a, b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = 0. If Rolle's Theorem cannot be applied, explain why not.
(similar to p.216 #11-26)

$$f(x) = x^2 - 4x - 32, \quad [-4,8]$$

5. Determine whether Rolle's Theorem can be applied to f on the closed interval [a, b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = 0. If Rolle's Theorem cannot be applied, explain why not.
(similar to p.216 #11-26)

$$f(x) = \frac{x^2 - 4}{x + 5} \quad [-2, 2]$$

6. Determine whether Rolle's Theorem can be applied to f on the closed interval [a, b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = 0. If Rolle's Theorem cannot be applied, explain why not.
(similar to p.216 #11-26)

$$f(x) = \sin 2x \quad [-\pi, \pi]$$

The Mean Value Theorem

If f is continuous on the closed interval [a, b] and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

7. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = [f(b) – f(a)] / (b – a). If the Mean Value Theorem cannot be applied, explain why not.
(similar to p.217 #43-56)

$$f(x) = 3x^2 \quad [0, 1]$$

8. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = [f(b) – f(a)] / (b – a). If the Mean Value Theorem cannot be applied, explain why not.
(similar to p.217 #43-56)

$$f(x) = x^3 - 2x \quad [0, 5]$$

9. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = [f(b) – f(a)] / (b – a). If the Mean Value Theorem cannot be applied, explain why not.
(similar to p.217 #43-56)

$$f(x) = |5x + 2| \quad [-3, 4]$$

10. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = [f(b) – f(a)] / (b – a). If the Mean Value Theorem cannot be applied, explain why not. (similar to p.217 #43-56) NEXT TIME SPRING 2013

$$f(x) = \sqrt{5+x} \quad [-4,4]$$

11. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = [f(b) – f(a)] / (b – a). If the Mean Value Theorem cannot be applied, explain why not. (similar to p.217 #43-56)

$$f(x) = \cos x \quad \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

12. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c) = [f(b) – f(a)] / (b – a). If the Mean Value Theorem cannot be applied, explain why not. (similar to p.217 #43-56)

$$f(x) = e^{5x} \quad [0,3]$$