5. $f(x)=\sqrt{x+2}$ $\quad (\underset{x}{7},\underset{y}{3})$

FIND EQUATION OF TANGENT LINE

① FIND DERIVATIVE

ⓐ FIND $f(x)$

$f(x)=\sqrt{x+2}$

ⓑ FIND $f(x+h)$

$f(x+h)=\sqrt{x+h+2}$

ⓒ PLUG $f(x)$ AND $f(x+h)$ INTO FORMULA AND FIND LIMIT

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\cdot\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$$=\lim_{h\to 0}\frac{x+h+2-(x+2)}{h(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$=\lim_{h\to 0}\frac{x+h+2-x-2}{h(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$$=\frac{1}{\sqrt{x+0+2}+\sqrt{x+2}}$$

$$f'(x)=\frac{1}{2\sqrt{x+2}}$$

② CHANGE $f'(x)$ TO m AND PLUG IN X PART OF POINT

$$f'(x)=\frac{1}{2\sqrt{x+2}}$$

$$m=\frac{1}{2\sqrt{7+2}}$$

$$m=\frac{1}{2\sqrt{9}}$$

$$m=\frac{1}{2(3)}$$

$$m=\frac{1}{6}$$

③ PLUG IN m FROM STEP ② AND GIVEN POINT FOR x, y INTO $y=mx+b$ AND SOLVE FOR b

$y=mx+b$

$3=\frac{1}{6}(7)+b$

$3=\frac{7}{6}+b$

$3-\frac{7}{6}=b$

$b=\frac{11}{6}$

④ WRITE ANSWER

$y=mx+b$

$$\boxed{y=\frac{1}{6}x+\frac{11}{6}}$$