Calculus I
Chapter 3 Test

Instructions: In all the problems, find the derivative, get rid of any negative exponents and complex fractions appropriately and then stop, don't simplify ANY FURTHER!

In problems 1-3, find the derivative

1. $y = 2x^4 - 3x^2 - 7x - 1$

$y' = 8x^3 - 6x - 7$

2. $y = 2x^{-5} - x^{-3} + 4$

$y' = -10x^{-6} + 3x^{-4}$

$y' = \frac{-10}{x^6} + \frac{3}{x^4}$

3. $y = \frac{2}{3x^2} + \frac{5}{x} - 8x$

$y = \frac{2}{3}x^{-2} + 5x^{-1} - 8x$

$y' = \frac{-4}{3}x^{-3} - 5x^{-2} - 8$

$y' = \frac{-4}{3x^3} - \frac{5}{x^2} - 8$

Find the second derivative:

4. $y = x^5 - 2x^3 + x^2 - x - 2$

$y' = 5x^4 - 6x^2 + 2x - 1$

$y'' = 20x^3 - 12x + 2$

In problems 5 and 6, find y' using the product rule

5. $y = (x^3 - 5x^2 + 2x)(x^2 - 7x)$

$p = x^3 - 5x^2 + 2x$

$p' = 3x^2 - 10x + 2$

$q = x^2 - 7x$

$q' = 2x - 7$

$Formula: p'q + pq'$

$y' = (3x^2 - 10x + 2)(x^2 - 7x) + (x^3 - 5x^2 + 2x)(2x - 7)$

6. $y = \left(\frac{-3}{x^2} - 1\right)\left(x^5 - \frac{1}{x} + 5\right)$

$y = (-3x^{-2} - 1)(x^5 - x^{-1} + 5)$

$p = -3x^{-2} - 1$

$p' = 6x^{-3}$

$q = x^5 - x^{-1} + 5$

$q' = 5x^4 + x^{-2}$

$Formula: p'q + pq'$

$y' = (6x^{-3})(x^5 - x^{-1} + 5) + (-3x^{-2} - 1)(5x^4 + x^{-2})$

$y' = \left(\frac{6}{x^3}\right)\left(x^5 - \frac{1}{x} + 5\right) + \left(\frac{-3}{x^2} - 1\right)\left(5x^4 + \frac{1}{x^2}\right)$

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Find the derivative of the function

7. $y = \dfrac{x^2 + 4x - 2}{x^3 - 7x^2}$

$$p = x^2 + 4x - 2$$
$$p' = 2x + 4$$
$$q = x^3 - 7x^2$$
$$q' = 3x^2 - 14x$$
$$Formula : \frac{p'q - pq'}{q^2}$$
$$y' = \frac{(2x+4)(x^3 - 7x^2) - (x^2 + 4x - 2)(3x^2 - 14x)}{(x^3 - 7x^2)^2}$$

8. Find an equation for the tangent to the curve $y = \dfrac{1}{x^2 - 2x}$ at the point (1, -1)

$$y = (x^2 - 2x)^{-1}$$
$$y' = -(x^2 - 2x)^{-2} \cdot (2x - 2)$$
$$y' = \frac{-(2x - 2)}{(x^2 - 2x)^2}$$
$$m = \frac{-\left(2(1) - 2\right)}{\left((1)^2 - 2(1)\right)^2}$$
$$m = 0$$
$$then$$
$$y = mx + b$$
$$-1 = 0(1) + b$$
$$-1 = 0 + b$$
$$-1 = b$$
$$answer :$$
$$y = mx + b$$
$$y = 0x - 1$$
$$y = -1$$

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Find the derivative:

9. $y=\sin^2(5x)-\dfrac{3x}{\cos x}$

$$fraction:$$
$$p=3x$$
$$p'=3$$
$$q=\cos x$$
$$q'=-\sin x$$
$$formula:\frac{p'q-pq'}{q^2}$$
$$y=[\sin(5x)]^2-\frac{3x}{\cos x}$$
$$y'=2[\sin(5x)]^1\cdot\frac{d}{dx}[\sin(5x)]-\frac{3\cos x-3x(-\sin x)}{(\cos x)^2}$$
$$y'=2[\sin(5x)]\cdot\cos(5x)\cdot\frac{d}{dx}(5x)-\frac{3\cos x+3x\sin x}{\cos^2 x}$$
$$y'=2[\sin(5x)]\cos(5x)\cdot 5-\frac{3\cos x+3x\sin x}{\cos^2 x}$$
$$y=10\sin(5x)\cos(5x)-\frac{3\cos x+3x\sin x}{\cos^2 x}$$

Find the derivative

10. $q=\sqrt[5]{x-x^3}$

$$q=\left(x-x^3\right)^{\frac{1}{5}}$$
$$q'=\frac{1}{5}\left(x-x^3\right)^{\frac{1}{5}-1}\cdot\frac{d}{dx}\left(x-x^3\right)$$
$$q'=\frac{1}{5}\left(x-x^3\right)^{\frac{-4}{5}}(1-3x^2)$$
$$q'=\frac{1-3x^2}{5\left(x-x^3\right)^{\frac{4}{5}}}$$

11. $h(x)=\left(\frac{\sin^2 x}{\cos x}\right)^{\frac{1}{2}}$

$$h'(x)=\frac{1}{2}\left(\frac{\sin^2 x}{\cos x}\right)^{\frac{1}{2}-1}\cdot\frac{d}{dx}\left(\frac{\sin^2 x}{\cos x}\right)$$
$$p=\sin^2 x=(\sin x)^2$$
$$p'=2(\sin x)^1\cdot\cos x=2\sin x\cos x$$
$$q=\cos x$$
$$q'=-\sin x$$
$$formula:\frac{p'q-pq'}{q^2}$$
$$h'(x)=\frac{1}{2}\left(\frac{\sin^2 x}{\cos x}\right)^{\frac{-1}{2}}\cdot\frac{2\sin x\cos x\cos x-\sin^2 x(-\sin x)}{(\cos x)^2}$$
$$h'(x)=\frac{1}{2}\left(\frac{\cos x}{\sin^2 x}\right)^{\frac{1}{2}}\cdot\frac{2\sin x\cos^2 x+\sin^3 x}{\cos^2 x}$$

Use implicit differentiation to find dy/dx

12. $3xy-y^3=2y+7$

$$\frac{d}{dx}(3xy)-\frac{d}{dx}(y^3)=\frac{d}{dx}(2y)+\frac{d}{dx}(7)$$
$$p=3x$$
$$p'=3$$
$$q=y$$
$$q'=y'$$
$$formula: p'q+pq'$$
$$3y+3xy'-3y^2y'=2y'$$
$$3y=2y'-3xy'+3y^2y'$$
$$3y=y'(2-3x+3y^2)$$
$$\frac{3y}{2-3x+3y^2}=y'$$

13. $\frac{4x-1}{5x} = x^5 - y^4$

$$\frac{4x}{5x} - \frac{1}{5x} = x^5 - y^4$$

$$\frac{4}{5} - \frac{1}{5}x^{-1} = x^5 - y^4$$

$$\frac{d}{dx}\left(\frac{4}{5}\right) - \frac{d}{dx}\left(\frac{1}{5}x^{-1}\right) = \frac{d}{dx}\left(x^5\right) - \frac{d}{dx}\left(y^4\right)$$

$$0 - \frac{-1}{5}x^{-2} = 5x^4 - 4y^3y'$$

$$\frac{1}{5x^2} = 5x^4 - 4y^3y'$$

$$5x^2\left(\frac{1}{5x^2}\right) = 5x^2\left(5x^4\right) + 5x^2\left(-4y^3y'\right)$$

$$1 = 25x^6 - 20x^2y^3y'$$

$$20x^2y^3y' = 25x^6 - 1$$

$$y' = \frac{25x^6 - 1}{20x^2y^3}$$

14. $\sin(xy) - 4x = y$

$$\frac{d}{dx}\left[\sin(xy)\right] + \frac{d}{dx}(-4x) = \frac{d}{dx}(y)$$

$$\cos(xy) \cdot \frac{d}{dx}(xy) - 4 = 1 \cdot y'$$

$$p = x$$

$$p' = 1$$

$$q = y$$

$$q' = y'$$

$$formula: p'q + pq'$$

$$\cos(xy) \cdot (1 \cdot y + xy') - 4 = y'$$

$$\cos(xy) \cdot (y + xy') - 4 = y'$$

$$y\cos(xy) + xy'\cos(xy) - 4 = y'$$

$$y\cos(xy) - 4 = y' - xy'\cos(xy)$$

$$y\cos(xy) - 4 = y'(1 - x\cos(xy))$$

$$\frac{y\cos(xy) - 4}{1 - x\cos(xy)} = y'$$

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Use log differentiation to find the derivative of y

15. $y=\sqrt[4]{x(x-7)(x+3)}$

$$y=[x(x-7)(x+3)]^{\frac{1}{4}}$$
$$\ln y=\ln[x(x-7)(x+3)]^{\frac{1}{4}}$$
$$\ln y=\frac{1}{4}\ln[x(x-7)(x+3)]$$
$$\ln y=\frac{1}{4}\ln x+\frac{1}{4}\ln(x-7)+\frac{1}{4}\ln(x+3)$$
$$\frac{d}{dx}(\ln y)=\frac{d}{dx}\left(\frac{1}{4}\ln x\right)+\frac{d}{dx}\left(\frac{1}{4}\ln(x-7)\right)+\frac{d}{dx}\left(\frac{1}{4}\ln(x+3)\right)$$
$$\frac{1}{y}y'=\frac{1}{4}\cdot\frac{1}{x}+\frac{1}{4}\cdot\frac{1}{x-7}+\frac{1}{4}\cdot\frac{1}{x+3}$$
$$y'=y\left[\frac{1}{4x}+\frac{1}{4(x-7)}+\frac{1}{4(x+3)}\right]$$
$$y'=\sqrt[4]{x(x-7)(x+3)}\left[\frac{1}{4x}+\frac{1}{4(x-7)}+\frac{1}{4(x+3)}\right]$$

Find the derivative

16. $y=\ln(2x^3)$

$$y=\ln 2+\ln x^3$$
$$y=\ln 2+3\ln x$$
$$y'=0+3\cdot\frac{1}{x}$$
$$y'=\frac{3}{x}$$

17. $y = 4x^2 e^{4x} - 7x$

$p = 4x^2$
$p' = 8x$
$q = e^{4x}$
$q' = 4e^{4x}$
$formula : p'q + pq'$
$y' = 8xe^{4x} + 4x^2 4e^{4x} - 7$
$y' = 8xe^{4x} + 16x^2 e^{4x} - 7$

18. $y = e^{x^2 - x} + 3x - \sin x$

$y' = e^{x^2 - x} \cdot (2x - 1) + 3 - \cos x$

19. $y = e^{4x - x^2}$

$y = e^{4x - x^2} \cdot (4 - 2x)$

20. $y = (x^3 - 4x + 2)e^{x^2 - 2x}$

$p = x^3 - 4x + 2$
$p' = 3x^2 - 4$
$q = e^{x^2 - 2x}$
$q' = e^{x^2 - 2x}(2x - 2)$
$formula : p'q + pq'$
$y' = (3x^2 - 4)e^{x^2 - 2x} + (x^3 - 4x + 2)e^{x^2 - 2x}(2x - 2)$