

$$4. \int \frac{5x}{x+3} dx$$

$$u = x+3 \quad du = 1 \cdot dx$$

$$u-3 = x$$

so

$$= 5 \int \frac{u-3}{u} du$$

$$= 5 \int \left(\frac{u}{u} - \frac{3}{u} \right) du$$

$$= 5 \int \left(1 - \frac{3}{u} \right) du$$

$$= 5 \left[u - 3 \ln|u| \right] + C$$

$$= 5 \left[x+3 - 3 \ln|x+3| \right] + C$$

$$= 5x + 15 - 15 \ln|x+3| + C$$

$$= \boxed{5x - 15 \ln|x+3| + C}$$

$$5. \int \frac{\cos x}{\sqrt{\sin x}} dx$$

$$u = \sin x \quad du = (\cos x) dx$$

$$= \int \frac{1}{\sqrt{u}} du$$

$$= \int \frac{1}{u^{1/2}} du$$

$$= \int u^{-1/2} du$$

$$= \frac{u^{-1/2+1}}{-1/2+1} + C$$

$$= \frac{u^{1/2}}{1/2} + C$$

$$= \frac{2}{1} \cdot u^{1/2} + C$$

$$= 2u^{1/2} + C$$

$$= 2\sqrt{u} + C$$

$$= \boxed{2\sqrt{\sin x} + C}$$

$$6. \int \frac{8}{5e^x - 3} dx$$

$$= 8 \int \frac{1}{\frac{5}{3}e^x - \frac{3}{3}} dx$$

$$= 8 \cdot \frac{1}{3} \int \frac{1}{\frac{5}{3}e^x + 1} dx$$

$$= -\frac{8}{3} \int \frac{-\frac{5}{3}e^x + 1 + \frac{5}{3}e^x}{-\frac{5}{3}e^x + 1} dx$$

$$= -\frac{8}{3} \left[\int \frac{-\frac{5}{3}e^x + 1}{-\frac{5}{3}e^x + 1} dx + \int \frac{\frac{5}{3}e^x}{-\frac{5}{3}e^x + 1} dx \right]$$

$$= -\frac{8}{3} \left[\int 1 dx - \int \frac{-\frac{5}{3}e^x}{-\frac{5}{3}e^x + 1} dx \right]$$

$$= -\frac{8}{3} \left[\int 1 dx - \int \frac{-\frac{5}{3}e^x}{-\frac{5}{3}e^x + 1} dx \right]$$

$$u = -\frac{5}{3}e^x + 1 \quad du = -\frac{5}{3}e^x dx$$

$$= -\frac{8}{3} \left[x - \int \frac{1}{u} du \right]$$

$$= -\frac{8}{3} \left[x - \ln|u| \right] + C$$

$$= -\frac{8}{3} \left[x - \ln \left| -\frac{5}{3}e^x + 1 \right| \right] + C$$

$$= \boxed{-\frac{8}{3}x + \frac{8}{3} \ln \left| -\frac{5}{3}e^x + 1 \right| + C}$$