7. $$\int \frac{1+\sin x}{\cos^2 x}\,dx$$

$$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right) dx$$

$$= \int \left(\sec^2 x + \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\right) dx$$

$$= \int (\sec^2 x + \tan x \sec x)\,dx$$

$$= \tan x + \sec x + C$$

8. $$\int \frac{1}{\sin x - 1}\,dx$$

$$= \int \left(\frac{1}{\sin x - 1}\cdot\frac{\sin x + 1}{\sin x + 1}\right) dx$$

$$= \int \frac{\sin x + 1}{\sin^2 x - 1}\,dx$$

RECALL

$\sin^2 + \cos^2 = 1$

$\sin^2 - 1 = -\cos^2$

$$= \int \frac{\sin x + 1}{-\cos^2 x}$$

$$= \int \frac{\sin x}{-\cos^2 x}\,dx + \int \frac{1}{-\cos^2 x}\,dx$$

$$= -\int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx - \int \sec^2 x\,dx$$

$$= -\int \tan x \sec x\,dx - \int \sec^2 x\,dx$$

$$= -\sec x - \tan x + C$$

9. $$\int \frac{1}{(x-1)\sqrt{9x^2-18x+4}}\,dx$$

$$= \int \frac{1}{(x-1)\sqrt{9(x^2-2x+\frac{4}{9})}}\,dx$$

$$= \int \frac{1}{(x-1)\cdot 3\sqrt{x^2-2x+\frac{4}{9}}}\,dx \qquad \left(-2\cdot\tfrac{1}{2}\right)^2 \quad (-1)^2 \quad 1$$

$$= \frac{1}{3}\int \frac{1}{(x-1)\sqrt{x^2-2x+1-1+\frac{4}{9}}}\,dx$$

$$= \frac{1}{3}\int \frac{1}{(x-1)\sqrt{(x-1)^2-\frac{5}{9}}}\,dx$$

$$= \frac{1}{3}\int \frac{1}{(x-1)\sqrt{(x-1)^2-\left(\sqrt{\frac{5}{9}}\right)^2}}\,dx$$

$u = x-1 \qquad a = \sqrt{\frac{5}{9}} \qquad du = 1\cdot dx$

$$= \frac{1}{3}\int \frac{1}{u\sqrt{u^2-a^2}}\,du$$

$$= \frac{1}{3}\cdot\frac{1}{a}\,\mathrm{arcsec}\,\frac{|u|}{a} + C$$

$$= \frac{1}{3}\cdot\frac{1}{\sqrt{\frac{5}{9}}}\,\mathrm{arcsec}\,\frac{|x-1|}{\sqrt{\frac{5}{9}}} + C$$

$$= \frac{1}{3}\cdot\frac{1}{\frac{\sqrt{5}}{\sqrt{9}}}\,\mathrm{arcsec}\,\frac{|x-1|}{\frac{\sqrt{5}}{\sqrt{9}}} + C$$

$$= \frac{1}{3}\cdot\frac{1}{\frac{\sqrt{5}}{3}}\,\mathrm{arcsec}\,\frac{|x-1|}{\frac{\sqrt{5}}{3}} + C$$

$$= \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{\sqrt{5}}\,\mathrm{arcsec}\,\frac{3}{\sqrt{5}}|x-1| + C$$

$$= \frac{\sqrt{5}}{5}\,\mathrm{arcsec}\left(\frac{3\sqrt{5}}{5}|x-1|\right) + C$$