4. $\int (e^{3x} \sin x)\,dx$

$$\int \underbrace{\sin x}_{u}\ \underbrace{e^{3x}\,dx}_{dv}$$

$uv - \int v\,du$

$$\int \sin x\, e^{3x}\,dx = \sin x \cdot \tfrac{1}{3}e^{3x} - \int \tfrac{1}{3}e^{3x}\cos x\,dx$$

$u = \sin x$ $dv = e^{3x}\,dx$

$du = \cos x\,dx$ $v = \int e^{3x}\,dx$ $w = 3x$, $dw = 3\,dx$

$= \frac{1}{3}\int e^{3x}\,3\,dx$

$= \frac{1}{3}\int e^{w}\,dw$

$= \frac{1}{3}e^{w}$

$= \frac{1}{3}e^{3x}$

PASS 1

$$\int \sin x\, e^{3x}\,dx = \tfrac{1}{3}e^{3x}\sin x - \tfrac{1}{3}\int \underbrace{\cos x}_{u}\ \underbrace{e^{3x}\,dx}_{dv}$$

"$uv - \int v\,du$"

$u = \cos x$ $dv = e^{3x}\,dx$

$du = -\sin x\,dx$ $v = \int e^{3x}\,dx$

$v = \frac{1}{3}e^{3x}$

PASS 2

$$\int \sin x\, e^{3x}\,dx = \tfrac{1}{3}e^{3x}\sin x - \tfrac{1}{3}\left[\cos x \cdot \tfrac{1}{3}e^{3x} - \int \tfrac{1}{3}e^{3x}(-\sin x\,dx)\right]$$

$$\int \sin x\, e^{3x}\,dx = \tfrac{1}{3}e^{3x}\sin x - \tfrac{1}{3}\left[\tfrac{1}{3}e^{3x}\cos x + \tfrac{1}{3}\int \sin x\, e^{3x}\,dx\right]$$

$$\int \sin x\, e^{3x}\,dx = \tfrac{1}{3}e^{3x}\sin x - \tfrac{1}{9}e^{3x}\cos x - \tfrac{1}{9}\int \sin x\, e^{3x}\,dx$$

$$1\int \sin x\, e^{3x}\,dx + \tfrac{1}{9}\int \sin x\, e^{3x}\,dx = \tfrac{1}{3}e^{3x}\sin x - \tfrac{1}{9}e^{3x}\cos x$$

$$\tfrac{10}{9}\int \sin x\, e^{3x}\,dx = \tfrac{1}{3}e^{3x}\sin x - \tfrac{1}{9}e^{3x}\cos x$$

$$\cancel{\tfrac{9}{10}} \cdot \cancel{\tfrac{10}{9}}\int \sin x\, e^{3x}\,dx = \tfrac{9}{10}\left[\tfrac{1}{3}e^{3x}\sin x - \tfrac{1}{9}e^{3x}\cos x\right]$$

$$\int \sin x\, e^{3x}\,dx = \tfrac{3}{10}e^{3x}\sin x - \tfrac{1}{10}e^{3x}\cos x + C$$