9. $\int \tan^5(9x)\,dx$

$\int \tan^4(9x)\tan(9x)\,dx$ (SAVE)

RECALL

$1+\tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$\int \tan^2(9x)\tan^2(9x)\tan(9x)\,dx$

$\int (\sec^2 9x - 1)(\sec^2 9x - 1)\tan(9x)\,dx$

$\int (\sec^4 9x - \sec^2 9x - \sec^2 9x + 1)\tan(9x)\,dx$

$\int (\sec^4 9x - 2\sec^2 9x + 1)\tan(9x)\,dx$

$\int \left(\sec^4(9x)\tan(9x) - 2\sec^2(9x)\tan(9x) + \tan(9x)\right)dx$

$\int \left((\sec 9x)^3 \sec 9x \tan 9x - 2(\sec 9x)\sec 9x \tan 9x + \tan(9x)\right)dx$

$\int (\sec 9x)^3 \sec 9x \tan 9x\,dx - 2\int (\sec 9x)^1 \sec 9x \tan 9x\,dx + \int \tan(9x)\,dx$

$u = \sec 9x \quad du = \sec 9x \tan 9x \cdot 9\,dx$; $w = 9x \quad dw = 9\,dx$

$\frac{1}{9}\int (\sec 9x)^3 \cdot 9\sec 9x \tan 9x\,dx - \frac{2}{9}\int (\sec 9x)^1 \cdot 9\sec 9x \tan 9x\,dx + \frac{1}{9}\int \tan(9x)\cdot 9\,dx$

$\frac{1}{9}\int u^3\,du - \frac{2}{9}\int u^1\,du + \frac{1}{9}\int \tan w\,dw$

$\frac{1}{9}\cdot\frac{1}{4}u^4 - \frac{2}{9}\cdot\frac{1}{2}u^2 + \frac{1}{9}\left(-\ln|\cos w|\right) + C$

$\frac{1}{36}\sec^4(9x) - \frac{1}{9}\sec^2(9x) - \frac{1}{9}\ln|\cos 9x| + C$