Form: $y' + P(x)\,y = Q(x)$

5. $xy' + 3y = \sin x \quad , \quad x > 0$

① $\dfrac{xy'}{x} + \dfrac{3y}{x} = \dfrac{\sin x}{x}$

$y' + \underbrace{\dfrac{3}{x}}_{P(x)} y = \dfrac{\sin x}{x}$

② I.F.

$\mu(x) = e^{\int P(x)\,dx}$

$= e^{\int \frac{3}{x}\,dx}$

$= e^{3\ln x}$

$= e^{\ln x^3}$

$= x^3$

③ $x^3\left(y' + \dfrac{3}{x}y\right) = x^3\left(\dfrac{\sin x}{x}\right)$

$\underbrace{x^3}_{Q}\,\underbrace{y'}_{P'} + \underbrace{3x^2}_{Q'}\,\underbrace{y}_{P} = x^2 \sin x$

④ $\dfrac{d}{dx}\left[yx^3\right] = x^2 \sin x$

⑤ $\displaystyle\int \frac{d}{dx}\left[yx^3\right] dx = \int x^2 \sin x \, dx$ "PARTS"

$yx^3 =$

S	D	I
+	x^2	$\sin x$
−	$2x$	$-\cos x$
+	2	$-\sin x$
−	0	$-(-\cos x) = \cos x$

$yx^3 = -x^2\cos x + 2x\sin x + 2\cos x + C$

$\dfrac{yx^3}{x^3} = \dfrac{-x^2\cos x}{x^3} + \dfrac{2x\sin x}{x^3} + \dfrac{2\cos x}{x^3} + \dfrac{C}{x^3}$

$$\boxed{y = \frac{-\cos x}{x} + \frac{2\sin x}{x^2} + \frac{2\cos x}{x^3} + \frac{C}{x^3}}$$