$$\boxed{\text{Form: } y' + P(x)\,y = Q(x)}$$

8. $y' + \underbrace{1}_{P(x)}y = \sin(2x)$

2 I.F.

$$\mu(x) = e^{\int P(x)\,dx} = e^{\int 1\,dx} = e^{x}$$

3 $e^{x}(y' + 1y) = e^{x}\sin(2x)$

4 $\underbrace{e^{x}}_{Q}\underbrace{y'}_{P'} + \underbrace{e^{x}}_{Q'}\underbrace{y}_{P} = e^{x}\sin(2x)$

$$\frac{d}{dx}\left[y e^{x}\right] = e^{x}\sin(2x)$$

5 $\int \frac{d}{dx}\left[y e^{x}\right]dx = \boxed{\int e^{x}\sin(2x)\,dx}$

$$\int u\,dv = uv - \int v\,du$$

$y e^{x} =$ ________________

$$\int e^{x}\sin(2x)\,dx = \int \underbrace{\sin(2x)}_{u}\,\underbrace{e^{x}\,dx}_{dv}$$

$$\int e^{x}\sin(2x)\,dx = \sin(2x)e^{x} - \int e^{x}\cdot 2\cos(2x)\,dx$$

$$\int e^{x}\sin(2x)\,dx = e^{x}\sin(2x) - 2\int \underbrace{\cos(2x)}_{u}\,\underbrace{e^{x}\,dx}_{dv}$$

$$\int e^{x}\sin(2x)\,dx = e^{x}\sin(2x) - 2\left[\cos(2x)e^{x} - \int e^{x}(-\sin(2x)\cdot 2)\,dx\right]$$

$$\int e^{x}\sin(2x)\,dx = e^{x}\sin(2x) - 2e^{x}\cos(2x) - 4\int e^{x}\sin(2x)\,dx$$

$$1\int e^{x}\sin(2x)\,dx + 4\int e^{x}\sin(2x)\,dx = e^{x}\sin(2x) - 2e^{x}\cos(2x)$$

$$5\int e^{x}\sin(2x)\,dx = e^{x}\sin(2x) - 2e^{x}\cos(2x)$$

$$\boxed{\int e^{x}\sin(2x)\,dx = \frac{1}{5}e^{x}\sin(2x) - \frac{2}{5}e^{x}\cos(2x)}$$

$$\frac{y e^{x}}{e^{x}} = \frac{\frac{1}{5}e^{x}\sin(2x)}{e^{x}} - \frac{\frac{2}{5}e^{x}\cos(2x)}{e^{x}} + \frac{C}{e^{x}}$$

$$y = \frac{1}{5}\sin(2x) - \frac{2}{5}\cos(2x) + Ce^{-x}$$