$$\boxed{\text{Form: } y' + P(x)\,y = Q(x)}$$

10. $(y - x\sin x^2)\,dx + x\,dy = 0$

① $\dfrac{(y - x\sin x^2)\,dx}{dx} + \dfrac{x\,dy}{dx} = \dfrac{0}{dx}$

$(y - x\sin x^2) + \dfrac{x\,dy}{dx} = 0$

$x \cdot \dfrac{dy}{dx} + y = x\sin x^2$

$\dfrac{x \cdot \frac{dy}{dx}}{x} + \dfrac{y}{x} = \dfrac{x\sin x^2}{x}$

$y' + \underbrace{\tfrac{1}{x}}_{P(x)}\, y = \sin x^2$

② I.F.

$\mu(x) = e^{\int P(x)\,dx}$

$= e^{\int \frac{1}{x}\,dx}$

$= e^{\ln x}$

$= x$

③ $x\left(y' + \tfrac{1}{x}y\right) = x\sin x^2$

$\underbrace{x}_{Q}\,\underbrace{y'}_{P'} + \underbrace{1}_{Q'}\,\underbrace{y}_{P} = x\sin x^2$

④ $\dfrac{d}{dx}[yx] = x\sin x^2$

⑤ $\int \dfrac{d}{dx}[yx]\,dx = \int x\sin \underbrace{x^2}_{u}\,dx$

$u = x^2$

$du = 2x\,dx$

$yx = \frac{1}{2}\int 2x\sin x^2\,dx$

$yx = \frac{1}{2}\int \sin u\,du$

$yx = \frac{1}{2}(-\cos u) + C$

$yx = -\frac{1}{2}\cos x^2 + C$

$\dfrac{yx}{x} = -\dfrac{\cos x^2}{2x} + \dfrac{C}{x}$

$$\boxed{y = \frac{-\cos x^2}{2x} + \frac{C}{x}}$$