

Limits - Key

In problems 1-2, find the limit using the graph of $f(x)$

1. $\lim_{x \rightarrow 1} f(x) = 2$	2. $\lim_{x \rightarrow 1} f(x) = -2$
---	--

In problems 3-6, find the limit by filling out the table

3. $\lim_{x \rightarrow 3} (x^2 + 4x) = 21$

x	2.9	2.99	2.999	3	3.001	3.01	3.1
f(x)	20.01	20.9	20.99	21	21.01	21.1	22.01

4. $\lim_{x \rightarrow 1} \left(\frac{x-1}{x^2 - 3x + 2} \right) = -1$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
f(x)	-0.9091	-0.9901	-0.999	-1	-1.001	-1.01	-1.111

5. $\lim_{x \rightarrow 0} \left(\frac{\sqrt{x+4} - 2}{x} \right) = 0.25$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	0.25158	0.25016	0.25002	0.25	0.24998	0.24984	0.24846

6. $\lim_{x \rightarrow -3} \left(\frac{\frac{1}{3} - \frac{1}{x+3}}{x} \right) = DNE$

x	-3.1	-3.01	-3.001	-3	-2.999	-2.99	-2.9
f(x)	-3.333	-33.33	-333.3	DNE (does not exist)	333.33	33.333	3.3333

In problems 7-16, find the limit using direct substitution

7. $\lim_{x \rightarrow 2} 8 = 8$	8. $\lim_{x \rightarrow -5} (x^2 - 3x) = 40$
-----------------------------------	--

Limits - Key

9. $\lim_{x \rightarrow 4} \frac{\sqrt{x+3}}{10} = \frac{1}{2}$	10. $\lim_{x \rightarrow 3} \sqrt[3]{x^2 + 4x - 13} = 2$
11. $\lim_{x \rightarrow -1} (x^2 + 8x - 2) = -9$	12. $\lim_{x \rightarrow 2} \frac{x+3}{x-7} = -1$
13. $\lim_{x \rightarrow 1} (x^4 - 3x + 5) = 3$	14. $\lim_{x \rightarrow 7} x - 10 = 3$
15. $\lim_{x \rightarrow 3} \left(x^2 - \frac{1}{x} \right) = \frac{26}{3}$	16. $\lim_{x \rightarrow 2} \frac{\frac{1}{x-4} + 3}{\frac{1}{x+5}} = \frac{17}{2}$

In problems 17-25, find each limit using algebraic techniques

17. $\lim_{x \rightarrow 8} \frac{x-8}{x^2-64} = \frac{1}{16}$	18. $\lim_{x \rightarrow 2} \frac{x^2-5x+6}{x^2+2x-8} = \frac{-1}{6}$
19. $\lim_{x \rightarrow 5} \frac{2x^2+9x-5}{x^2+x-20} = \frac{11}{9}$	20. $\lim_{\Delta x \rightarrow 0} \frac{3(x+\Delta x)+4-(3x+4)}{\Delta x} = 3$
21. $\lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 + 7(x+\Delta x) - (x^2 + 7x)}{\Delta x} = 2x + 7$	22. $\lim_{x \rightarrow -6} \frac{\sqrt{x+7}-1}{x+6} = \frac{1}{2}$
23. $\lim_{x \rightarrow 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} = \frac{\sqrt{2}}{4}$	24. $\lim_{x \rightarrow 6} f(x) = DNE$ (does not exist)
25. $\lim_{x \rightarrow 2} f(x) = 11$	

In problems 26-27, use the graph to find the limit

26. $\lim_{x \rightarrow -3} \frac{1}{x+3} = DNE$	27. $\lim_{x \rightarrow 1} \frac{2}{x^2-3x+2} = DNE$
---	---

Limits - Key

In problems 28-30, use the graph to find the one-sided limit

28. $\lim_{x \rightarrow -1^-} \frac{ x+1 }{x+1} = -1$ and $\lim_{x \rightarrow -1^+} \frac{ x+1 }{x+1} = 1$	29. $\lim_{x \rightarrow 2^-} \frac{1}{x-2} = -\infty$
30. $\lim_{x \rightarrow -4^+} \frac{1}{x^2 + 5x + 4} = -\infty$	

In problems 31-32, use the graph to find the limit

$$(a) \lim_{x \rightarrow c^+} f(x) \quad (b) \lim_{x \rightarrow c^-} f(x) \quad (c) \lim_{x \rightarrow c} f(x)$$

31. (a) 1 (b) 1 (c) 1

32. (a) 4 (b) 1 (c) DNE