

$$1. \vec{r}(t) = 7t\vec{i} - 4t\vec{j} \quad [0, 5]$$

$$\vec{r}'(t) = 7\vec{i} - 4\vec{j}$$

$$\begin{aligned} \|\vec{r}'(t)\| &= \sqrt{7^2 + (-4)^2} \\ &= \sqrt{49 + 16} \\ &= \sqrt{65} \end{aligned}$$

$$S = \int_0^5 \sqrt{65} dt$$

$$S = [\sqrt{65}t]_0^5$$

$$S = \sqrt{65}(5) - \sqrt{65}(0)$$

$$S = 5\sqrt{65}$$

$$2. \vec{r}(t) = 4t^2\vec{i} - t^3\vec{j} \quad [0, 2]$$

$$\vec{r}'(t) = 8t\vec{i} - 3t^2\vec{j}$$

$$\begin{aligned} \|\vec{r}'(t)\| &= \sqrt{(8t)^2 + (-3t^2)^2} \\ &= \sqrt{64t^2 + 9t^4} \\ &= \sqrt{t^2(64 + 9t^2)} \\ &= t\sqrt{64 + 9t^2} \end{aligned}$$

$$S = \int_0^2 t\sqrt{64 + 9t^2} dt$$

$$u = 64 + 9t^2 \quad du = (18t)dt$$

$$S = \frac{1}{18} \int_0^2 18t(64 + 9t^2)^{\frac{1}{2}} dt$$

$$S = \frac{1}{18} \int_{t=0}^{t=2} u^{\frac{1}{2}} du$$

$$S = \frac{1}{18} \left[ \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_{t=0}^{t=2}$$

$$S = \frac{1}{18} \cdot \frac{2}{3} \left[ u^{\frac{3}{2}} \right]_{t=0}^{t=2}$$

$$S = \frac{1}{27} \left[ (64 + 9t^2)^{\frac{3}{2}} \right]_0^2$$

$$S = \frac{1}{27} \left[ (64 + 9(2)^2)^{\frac{3}{2}} - (64 + 9(0)^2)^{\frac{3}{2}} \right]$$

$$S = \frac{1}{27} \left[ 100^{\frac{3}{2}} - 64^{\frac{3}{2}} \right]$$

$$S = \frac{1}{27} \left[ (100^{\frac{1}{2}})^3 - (64^{\frac{1}{2}})^3 \right]$$

$$S = \frac{1}{27} \left[ (\sqrt{100})^3 - (\sqrt{64})^3 \right]$$

$$S = \frac{1}{27} [10^3 - 8^3]$$

$$S = \frac{1}{27} (1000 - 512)$$

$$S = \frac{1}{27} (488)$$

$$S = \frac{488}{27}$$

$\frac{64}{27}$