

$$6. \vec{r}(t) = t^2 \vec{i} + 5t \vec{j}, \quad t=1$$

$$\vec{r}'(t) = 2t \vec{i} + 5 \vec{j}$$

$$\|\vec{r}'(t)\| = \sqrt{(2t)^2 + 5^2} = \sqrt{4t^2 + 25}$$

$$\begin{aligned} \underline{t=1} \\ \|\vec{r}'(1)\| &= \sqrt{4(1)^2 + 25} \\ &= \sqrt{29} \end{aligned}$$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{2t \vec{i} + 5 \vec{j}}{\sqrt{4t^2 + 25}}$$

$$= \frac{2t}{\sqrt{4t^2 + 25}} \vec{i} + \frac{5}{\sqrt{4t^2 + 25}} \vec{j}$$

$$= \underbrace{2t}_P \underbrace{(4t^2 + 25)^{-1/2}}_Q \vec{i} + 5 (4t^2 + 25)^{-1/2} \vec{j}$$

$$P' = 2 \quad Q' = -\frac{1}{2} (4t^2 + 25)^{-3/2} \cdot 8t$$

$$= \frac{-4t}{(4t^2 + 25)^{3/2}}$$

$$P'Q + PQ'$$

$$\vec{T}'(t) = \left[2 (4t^2 + 25)^{-1/2} + 2t \left(\frac{-4t}{(4t^2 + 25)^{3/2}} \right) \right] \vec{i} + 5 \left(\frac{-4t}{(4t^2 + 25)^{3/2}} \right) \vec{j}$$

$$\vec{T}'(t) = \left[\frac{2}{(4t^2 + 25)^{1/2}} - \frac{8t^2}{(4t^2 + 25)^{3/2}} \right] \vec{i} + \frac{-20t}{(4t^2 + 25)^{3/2}} \vec{j}$$

$$\vec{T}'(t) = \left[\frac{2 \cdot (4t^2 + 25)^1}{(4t^2 + 25)^{1/2} \cdot (4t^2 + 25)^1} - \frac{8t^2}{(4t^2 + 25)^{3/2}} \right] \vec{i} + \frac{-20t}{(4t^2 + 25)^{3/2}} \vec{j}$$

$$\vec{T}'(t) = \left[\frac{8t^2 + 50}{(4t^2 + 25)^{3/2}} - \frac{8t^2}{(4t^2 + 25)^{3/2}} \right] \vec{i} + \frac{-20t}{(4t^2 + 25)^{3/2}} \vec{j}$$

$$\vec{T}'(t) = \frac{50}{(4t^2 + 25)^{3/2}} \vec{i} + \frac{-20t}{(4t^2 + 25)^{3/2}} \vec{j}$$