Arc Length and Curvature

Arc Length of a Space Curve

If C is a smooth curve given by
r(t) = x(t)**i** + y(t)**j** + z(t)**k**, on an interval [a, b], then the arc length of C on the interval is

$$s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$$

$$s = \int_a^b \|\mathbf{r}'(t)\|\,dt$$

1. Sketch the plane curve and find its length over the given interval (Similar to p.877 #1-6)

$$\boldsymbol{r}(t) = 7t\mathbf{i} - 4t\mathbf{j}\text{,}\ [0, 5]$$

2. Sketch the plane curve and find its length over the given interval (Similar to p.877 #1-6)

$$\boldsymbol{r}(t) = 4\mathrm{t}^2\mathbf{i} - \mathrm{t}^3\mathbf{j}\text{,}\ [0, 2]$$

3. Sketch the space curve and find its length over the given interval (Similar to p.877 #9-14)

$$\boldsymbol{r}(t) = 5\mathrm{t}\mathbf{i} + 2\mathrm{t}\mathbf{j} - 3\mathrm{t}\mathbf{k}\text{,}\ [0, 1]$$

4. Sketch the space curve and find its length over the given interval (Similar to p.877 #9-14)

$$\boldsymbol{r}(t) = < 2\mathrm{t}, 3\cos(\mathrm{t}), -3\sin(\mathrm{t}) >\text{,}\ \left[0, \frac{\pi}{2}\right]$$

Formulas for Curvature

If C is a smooth curve given by r(t), then the curvature K of C at t is given by

$$K = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

5. Find the curvature K of the plane curve at the given value of the parameter
(Similar to p.878 #25-30)

$$\boldsymbol{r}(t) = 5\mathrm{t}\mathbf{i} + 3\mathrm{t}\mathbf{j}, \mathrm{t} = 1$$

6. Find the curvature K of the plane curve at the given value of the parameter
(Similar to p.878 #25-30)

$$\boldsymbol{r}(t) = \mathrm{t}^2\mathbf{i} + 5\mathrm{t}\mathbf{j}, \mathrm{t} = 1$$

7. Find the curvature K of the plane curve at the given value of the parameter
(Similar to p.878 #31-40)

$$\boldsymbol{r}(t) = 3\mathbf{i} + 4\cos(\mathrm{t})\mathbf{j} + 4\sin(\mathrm{t})\mathbf{k}, \mathrm{t} = \pi$$

Curvature in Rectangular Coordinates

If C is the graph of a twice-differentiable function given by y = f(x), then the curvature K at the point (x, y) is given by

$$K = \frac{|y''|}{[1 + (y')^2]^{3/2}}$$

8. Find the curvature K of the plane curve at the point P
(Similar to p.878 #41-44)

$$\boldsymbol{r}(t) = 4\mathrm{t}\mathbf{i} + \mathrm{t}^2\mathbf{j}, (4, 1)$$