Center of Mass and Moments of Inertia

Definition of Mass of a Planar Lamina of Variable Density

If ρ is a continuous density function on the lamina corresponding to a plane region R, then the mass m of the lamina is given by:

$$m = \iint_R \rho(x, y)\, dA$$

1. Find the mass of the lamina described by the inequalities, given that its density is ρ(x,y) = xy (Hint: Some of the integrals are simpler in polar coordinates)
(Similar to p.1018 #1-4)

$$0 \le x \le 4, \quad 0 \le y \le 3$$

2. Find the mass of the lamina described by the inequalities, given that its density is ρ(x,y) = xy (Hint: Some of the integrals are simpler in polar coordinates)
(Similar to p.1018 #1-4)

$$0 \le x \le 2, \quad 0 \le y \le \sqrt{4 - x^2}$$

Moments and Center of Mass of a Variable Density Planar Lamina

Let ρ is a continuous density function on the planar lamina R. The moments of mass with respect to the x- and y-axes are:

$$M_x = \iint_R y\rho(x, y)\, dA \quad and \quad M_y = \iint_R x\rho(x, y)\, dA$$

If m is the mass of the lamina, then the center of mass is:

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$$

If R represents a simple plane region rather than a lamina, the point $(\bar{x}, \bar{y})$ is called the centroid of the region

3. Find the mass and center of mass of the lamina bounded by the graphs of the equations for the given density or densities (Hint: Some of the integrals are simpler in polar coordinates)
(Similar to p.1018 #11-22)

$$y = \sqrt{x}, y = 0, x = 4, \rho = ky$$

4. Find the mass and center of mass of the lamina bounded by the graphs of the equations for the given density or densities (Hint: Some of the integrals are simpler in polar coordinates) (Similar to p.1018 #11-22)

$$y = \frac{1}{x}, y = 0, x = 2, x = 6, \rho = kx^2$$

5. Find the mass and center of mass of the lamina bounded by the graphs of the equations for the given density or densities (Hint: Some of the integrals are simpler in polar coordinates) (Similar to p.1018 #11-22)

$$y = e^x, y = 0, x = 0, x = 1, \rho = kx$$

6. Find the mass and center of mass of the lamina bounded by the graphs of the equations for the given density or densities (Hint: Some of the integrals are simpler in polar coordinates) (Similar to p.1018 #11-22)

$$x^2 + y^2 = 9, x \geq 0, y \geq 0, \rho = k(x^2 + y^2)$$

Moments of Inertia

Moment of Inertia about x-axis: $I_x = \iint_R y^2 \rho(x, y)\, dA$

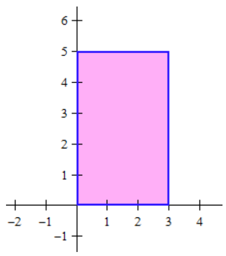
Moment of Inertia about y-axis: $I_y = \iint_R x^2 \rho(x, y)\, dA$

Polar Moment of Inertia: $I_0 = I_x + I_y$

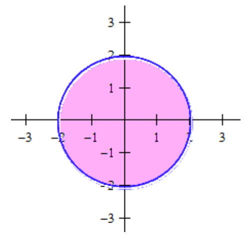
Radius of Gyration about x-axis: $\bar{\bar{y}} = \sqrt{\frac{I_x}{m}}$

Radius of Gyration about y-axis: $\bar{\bar{x}} = \sqrt{\frac{I_y}{m}}$

7. Find the moment(s) of inertia and $\bar{\bar{x}}$ and $\bar{\bar{y}}$. Assume that each lamina has a density of ρ = 1 gram per square centimeter (These regions are common shapes used in engineering) (Similar to p.1018 #27-32)



8. Find the moment(s) of inertia and $\bar{\bar{x}}$ and $\bar{\bar{y}}$. Assume that each lamina has a density of ρ = 1 gram per square centimeter (These regions are common shapes used in engineering) (Similar to p.1018 #27-32)



9. Find I_x, I_y, I_o, $\bar{\bar{x}}$ and $\bar{\bar{y}}$ for the lamina bounded by the graphs of the equations.
(Similar to p.1018 #33-40)

$$y = 9 - x^2, y = 0, x > 0, \rho = kx$$