

5. $y = e^x$, $y=0$, $x=0$, $x=1$, $\rho = kx$



$$m = \int_{x=0}^1 \int_{y=0}^{y=e^x} (kx) dy dx$$

$$m = \int_{x=0}^1 [kxy]_{y=0}^{y=e^x} dx$$

$$m = k \int_{x=0}^1 x[e^x - 0] dx$$

S	D	±
+	→ X	e^x
-	→ 1	e^x
+	0	e^x

$$m = k [xe^x - e^x]_{x=0}^{x=1}$$

$$m = k [1 \cdot e^1 - e^1 - (0 \cdot e^0 - e^0)]$$

$$m = k [-(-1)]$$

$m = k$

$$M_x = \int_{x=0}^1 \int_{y=0}^{y=e^x} y(kx) dy dx$$

$$M_x = k \int_{x=0}^1 \left[\frac{1}{2} y^2 x \right]_{y=0}^{y=e^x} dx$$

$$M_x = \frac{1}{2} k \int_{x=0}^1 x [(e^x)^2 - 0^2] dx$$

$$M_x = \frac{1}{2} k \int_{x=0}^1 x e^{2x} dx$$

S	D	±
+	→ X	$\frac{1}{2} e^{2x}$
-	→ 1	$\frac{1}{2} e^{2x}$
+	0	$\frac{1}{4} e^{2x}$

$$M_x = \frac{1}{2} k \left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right]_{x=0}^{x=1}$$

$$M_x = \frac{1}{2} k \left[\frac{1}{2} (1) e^{2(1)} - \frac{1}{4} e^{2(1)} \right]$$

$$- \left(\frac{1}{2} (0) e^{2(0)} - \frac{1}{4} e^{2(0)} \right)$$

$$M_x = \frac{1}{2} k \left[\frac{1}{2} e^2 - \frac{1}{4} e^2 + \frac{1}{4} (1) \right]$$

$$M_x = \frac{1}{2} k \left[\frac{2}{4} e^2 - \frac{1}{4} e^2 + \frac{1}{4} \right]$$

$$M_x = \frac{1}{2} k \left[\frac{1}{4} e^2 + \frac{1}{4} \right]$$

$$M_x = \frac{1}{8} k (e^2 + 1)$$

$$M_y = \int_{x=0}^1 \int_{y=0}^{y=e^x} x(kx) dy dx$$

$$M_y = k \int_{x=0}^1 \left[x^2 y \right]_{y=0}^{y=e^x} dx$$

$$M_y = k \int_{x=0}^1 x^2 [e^x - 0] dx$$

$$M_y = k \int_{x=0}^1 x^2 e^x dx$$

S	D	±
+	→ x^2	e^x
-	→ $2x$	e^x
+	→ 2	e^x
-	0	e^x

$$M_y = k [x^2 e^x - 2x e^x + 2e^x]_{x=0}^{x=1}$$

$$M_y = k [1^2 e^1 - 2(1) e^1 + 2e^1]$$

$$- (0^2 e^0 - 2(0) e^0 + 2e^0)$$

$$M_y = k [e - 2e + 2e - 2]$$

$$M_y = k [e - 2]$$

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right) = \left(\frac{k(e-2)}{k}, \frac{\frac{1}{8}k(e^2+1)}{k} \right) = \left(e-2, \frac{1}{8}(e^2+1) \right)$$