6. $x^2+y^2=9$, $x\geq 0$, $y\geq 0$ $\quad \rho = k(x^2+y^2)$

$r^2 = x^2+y^2$

$x = r\cos\theta$

$y = r\sin\theta$

m

$$m = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=4} (kr^2)\, r\, dr\, d\theta$$

$$m = k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=4} r^3\, dr\, d\theta$$

$$m = k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \left[\frac{1}{4}r^4\right]_{r=0}^{r=4} d\theta$$

$$m = \frac{1}{4}k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \left[4^4-0^4\right] d\theta$$

$$m = \frac{4^4}{4}k\int_{\theta=0}^{\theta=\frac{\pi}{2}} d\theta$$

$$m = 4^3 k\,[\theta]_{\theta=0}^{\theta=\frac{\pi}{2}}$$

$$m = 64k\left[\frac{\pi}{2}-0\right]$$

$$\boxed{m = 32\pi k}$$

M_x

$$M_x = \iint_R y\,\rho$$

$$M_x = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=4} (r\sin\theta \cdot kr^2)\, r\, dr\, d\theta$$

$$M_x = k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=4} r^4\sin\theta\, dr\, d\theta$$

$$M_x = k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \left[\frac{1}{5}r^5\sin\theta\right]_{r=0}^{r=4} d\theta$$

$$M_x = \frac{1}{5}k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin\theta\left[4^5-0^5\right] d\theta$$

$$M_x = \frac{1024}{5}k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin\theta\, d\theta$$

$$M_x = \frac{1024}{5}k\left[-\cos\theta\right]_{\theta=0}^{\theta=\frac{\pi}{2}}$$

$$M_x = \frac{1024}{5}k\left[-\cos\frac{\pi}{2}-(-\cos 0)\right]$$

$$M_x = \frac{1024}{5}k$$

M_y

$$M_y = \iint_R x\,\rho$$

$$M_y = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=4} r\cos\theta\, kr^2 \cdot r\, dr\, d\theta$$

$$M_y = k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=4} r^4\cos\theta\, dr\, d\theta$$

$$M_y = k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \left[\frac{1}{5}r^5\cos\theta\right]_{r=0}^{r=4} d\theta$$

$$M_y = \frac{1}{5}k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \cos\theta\left[4^5-0^5\right] d\theta$$

$$M_y = \frac{1024}{5}k\int_{\theta=0}^{\theta=\frac{\pi}{2}} \cos\theta\, d\theta$$

$$M_y = \frac{1024}{5}k\left[\sin\theta\right]_{\theta=0}^{\theta=\frac{\pi}{2}}$$

$$M_y = \frac{1024}{5}k\left[\sin\frac{\pi}{2}-\sin 0\right]$$

$$M_y = \frac{1024}{5}k$$

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

$$= \left(\frac{\frac{1024}{5}k}{32\pi k}, \frac{\frac{1024}{5}k}{32\pi k}\right)$$

$$= \left(\frac{1024}{160\pi}, \frac{1024}{160\pi}\right)$$

$$= \boxed{\left(\frac{32}{5\pi}, \frac{32}{5\pi}\right)}$$