

$$6. \quad w = x^2 + 3xy - 2y^2, \quad x = 2r + A, \quad y = 2r - \theta$$

$$\frac{\partial w}{\partial x} = 2x + 3y \quad \frac{\partial x}{\partial r} = 2 \quad \frac{\partial y}{\partial r} = 2$$

$$\frac{\partial w}{\partial y} = 3x - 4y \quad \frac{\partial x}{\partial A} = 1 \quad \frac{\partial y}{\partial A} = -1$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$= (2x + 3y)(2) + (3x - 4y)(2)$$

$$= 4x + 6y + 6x - 8y$$

$$= 10x - 2y$$

$$\text{plug in } x = 2r + A, \quad y = 2r - A$$

$$\frac{\partial w}{\partial r} = 10(2r + A) - 2(2r - A)$$

$$= 20r + 10A - 4r + 2A$$

$$= \boxed{16r + 12A}$$

$$\frac{\partial w}{\partial A} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial A} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial A}$$

$$= (2x + 3y)(1) + (3x - 4y)(-1)$$

$$= 2x + 3y - 3x + 4y$$

$$= -x + 7y$$

$$\text{plug in } x = 2r + A, \quad y = 2r - A$$

$$\frac{\partial w}{\partial A} = -(2r + A) + 7(2r - A)$$

$$= -2r - A + 14r - 7A$$

$$= \boxed{12r - 8A}$$

$$b) \quad w = x^2 + 3xy - 2y^2, \quad x = 2r + A, \quad y = 2r - \theta$$

$$w = (2r + \theta)^2 + 3(2r + \theta)(2r - \theta) - 2(2r - \theta)^2$$

$$w = 4r^2 + 4r\theta + \theta^2 + 3(4r^2 - \theta^2) - 2(4r^2 - 4r\theta + \theta^2)$$

$$w = 4r^2 + 4r\theta + \theta^2 + 12r^2 - 3\theta^2 - 8r^2 + 8r\theta - 2\theta^2$$

$$w = 8r^2 + 12r\theta - 4\theta^2$$

$$\frac{\partial w}{\partial r} = \boxed{16r + 12\theta}$$

$$\frac{\partial w}{\partial \theta} = \boxed{12r - 8\theta}$$