Chain Rules for Functions of Several Variables

1. Find dw/dt using the appropriate Chain Rule
(Similar to p.931 #1-4)

$$w = x^3 - 7y^2$$
$$x = t,\ y = 2t$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt}$$

2. Find dw/dt (a) by using the appropriate Chain Rule, and (b) by converting w to a function of t before differentiating
(Similar to p.931 #5-10)

$$w = x^2 y^2$$
$$x = e^t,\ y = e^{3t}$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt}$$

3. Find dw/dt (a) by using the appropriate Chain Rule, and (b) by converting w to a function of t before differentiating
(Similar to p.931 #5-10)

$$w = xy - xz + yz$$
$$x = t + 2,\ y = t - 3,\ z = t^2$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt}$$

4. Find d^2w/dt^2 using the appropriate Chain Rule. Evaluate d^2w/dt^2 at the given value of t
(Similar to p.931 #13-14)

$$w = \frac{x^3}{y^2}$$
$$x = t + 3,\ y = t^2, t = 1$$

5. Find ∂w/∂s and ∂w/∂t using the appropriate Chain Rule and evaluate each partial derivative at the given values of s and t
(Similar to p.931 #15-18)

$$w = x^3 + y^2$$
$$x = 4s + 2t,\ y = 5s - 3t$$
$$\text{Point}: s = 2, t = 1$$

6. Find ∂w/∂r and ∂w/∂θ (a) by using the appropriate Chain Rule and (b) by converting w to a function of r and θ before differentiating
(Similar to p.931 #19-22)

$$w = x^2 + 3xy - 2y^2$$
$$x = 2r + \theta,\ y = 2r - \theta$$

7. Find ∂w/∂s and ∂w/∂t by using the appropriate Chain Rule
(Similar to p.931 #23-26)

$$w = e^{x^2 y^3 z}$$
$$x = 3s + t,\ y = 2s - 4t,\ z = 5s$$

8. Differentiate implicitly to find dy/dx
(Similar to p.931 #27-30)

$$x^3 - 7xy - 4x + 3y^2 = 0$$

$$\frac{dy}{dx} = -\frac{F_x(x, y)}{F_y(x, y)}$$

9. Differentiate implicitly to find dy/dx
(Similar to p.931 #27-30)

$$\ln \sqrt[3]{3x^2 + y} - 7x = 0$$

10. Differentiate implicitly to find the first partial derivatives of z
(Similar to p.931 #31-38)

$$5x^2 y^3 z^4 = 2$$

$$\frac{\partial z}{\partial x} = -\frac{F_x(x, y, z)}{F_z(x, y, z)} \qquad \frac{\partial z}{\partial y} = -\frac{F_y(x, y, z)}{F_z(x, y, z)}$$

11. Differentiate implicitly to find the first partial derivatives of z
(Similar to p.931 #31-38)

$$\sin(x - z) - e^{yz} = 3y$$

12. Differentiate implicitly to find the first partial derivatives of w
(Similar to p.931 #39-42)

$$\tan(xyzw) - xy + 5wz = 3$$

$$\frac{\partial w}{\partial x} = -\frac{F_x}{F_w} \qquad \frac{\partial w}{\partial y} = -\frac{F_y}{F_w} \qquad \frac{\partial w}{\partial z} = -\frac{F_z}{F_w}$$