

# 4. (CONT.)

JACOBIAN

$$x = -\frac{1}{7}(u-4v)$$

$$y = -\frac{1}{7}(2u-v)$$

$$x = -\frac{1}{7}u + \frac{4}{7}v$$

$$y = -\frac{2}{7}u + \frac{1}{7}v$$

$$\begin{aligned} \frac{\partial(x,y)}{\partial(u,v)} &= \begin{vmatrix} -\frac{1}{7} & \frac{4}{7} \\ -\frac{2}{7} & \frac{1}{7} \end{vmatrix} = -\frac{1}{7}\left(\frac{1}{7}\right) - \left(-\frac{2}{7}\right)\left(\frac{4}{7}\right) \\ &= -\frac{1}{49} + \frac{8}{49} \\ &= \frac{7}{49} \\ &= \frac{1}{7} \end{aligned}$$

$$\iint_R 10xy$$

$$= \int_{v=-5}^{v=2} \int_{u=-6}^{u=2} 10 \cdot \left(-\frac{1}{7}(u-4v)\right) \cdot \left(-\frac{1}{7}\right)(2u-v) \cdot \frac{1}{7} du dv$$

$$= \frac{10}{7} \cdot \frac{1}{49} \int_{v=-5}^{v=2} \int_{u=-6}^{u=2} (u-4v)(2u-v) du dv$$

$$= \frac{10}{343} \int_{v=-5}^{v=2} \int_{u=-6}^{u=2} (2u^2 - 9uv + 4v^2) du dv$$

$$= \frac{10}{343} \int_{v=-5}^{v=2} \left[ \frac{2}{3}u^3 - \frac{9}{2}u^2v + 4uv^2 \right]_{u=-6}^{u=2} dv$$

$$= \frac{10}{343} \int_{v=-5}^{v=2} \left[ \frac{2}{3} \cdot 2^3 - \frac{9}{2} \cdot 2^2 \cdot v + 4 \cdot 2 \cdot v^2 - \left( \frac{2}{3}(-6)^3 - \frac{9}{2}(-6)^2 v + 4(-6)v^2 \right) \right] dv$$

$$= \frac{10}{343} \int_{v=-5}^{v=2} \left( \frac{16}{3} - 18v + 8v^2 - (-144 - 162v - 24v^2) \right) dv$$

$$\frac{6}{49} \cdot \frac{7}{343}$$

$$\frac{3}{343} \cdot \frac{6}{147}$$

$$\frac{72}{147}$$