Change of Variables: Jacobians

Definition: The Jacobian

If x = g(u, v) and y = h(u, v), then the Jacobian of x and y with respect to u and v is:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

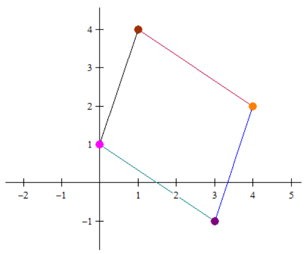
1. Find the Jacobian for the indicated change of variables
(Similar to p.1050 #1-8)

$$x = u + 3v, \; y = u - v$$

2. Find the Jacobian for the indicated change of variables
(Similar to p.1050 #1-8)

$$x = e^u \cos v, \; y = e^u \sin v$$

3. Determine the change of variables and sketch the image S in the uv-plane of the region R in the xy-plane
(Similar to p.1050 #1-8)



Change of Variables for Double Integrals

Let R be a vertically or horizontally simple region in the xy-plane, and let S be a vertically or horizontally simple region in the uv-plane. Let T from S to R be given by T(u, v) = (x, y) = (g(u,v), h(u,v)), where g and h have continuous first partial derivatives. Assume that T is one-to-one except possibly on the boundary of S. If f is continuous on R and the Jacobian is nonzero on S, then:

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(g(u,v),h(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right| \text{du dv}$$

4. Use a change of variables to find the volume of the solid region lying below the surface z = f(x, y) and above the plane region R
(Similar to p.1050 #21-29)

$f(x, y) = 10xy$

R : region bounded by the parallelogram with vertices : (-2,1), (2, 2), (-3, -1), (1, 0)