Change of Variables: Polar Coordinates

1. Evaluate the double integral $\int R \int f(r,\theta)\, dA$
(Similar to p.1009 #9-16)

$$\int_0^{\pi}\int_0^{\sin\theta} (\mathrm{r})\, dr\, \mathrm{d}\theta$$

2. Evaluate the double integral $\int R \int f(r,\theta)\, dA$
(Similar to p.1009 #9-16)

$$\int_0^{\pi/2}\int_0^{4} (\mathrm{r}^2\cos\theta)\, dr\, \mathrm{d}\theta$$

3. Evaluate the double integral $\int R \int f(r,\theta)\, dA$
(Similar to p.1009 #9-16)

$$\int_0^{\pi}\int_0^{2} (\mathrm{r}\sqrt{4-\mathrm{r}^2})\, dr\, \mathrm{d}\theta$$

Change of Variables to Polar Form

Let R be a plane region consisting of all points (x,y) = (r cos θ, r sin θ) satisfying the conditions $0 \le g_1(\theta) \le r \le g_2(\theta)$, $\alpha \le \theta \le \beta$, where $0 \le (\beta - \alpha) \le 2\pi$. If g_1 and g_2 are continuous on [α,β] and f is continuous on R, then

$$\iint_R f(x,y)\,\mathrm{dA} = \int_{\alpha}^{\beta}\int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta, r\sin\theta) r\,\mathrm{dr}\,\mathrm{d}\theta$$

4. Evaluate the iterated integral by converting to polar coordinates
(Similar to p.1009 #17-26)

$$\int_{-3}^{3}\int_0^{\sqrt{9-\mathrm{x}^2}} (\mathrm{x}^2+\mathrm{y}^2)\, dy\, \mathrm{dx}$$

5. Evaluate the iterated integral by converting to polar coordinates
(Similar to p.1009 #17-26)

$$\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)^{\frac{5}{2}}\, dy\, dx$$

6. Evaluate the iterated integral by converting to polar coordinates
(Similar to p.1009 #17-26)

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} (\cos\sqrt{x^2+y^2})\, dy\, dx$$

7. Use polar coordinates to set up and evaluate the double integral $\int R \int f(r,\theta)\, dA$
(Similar to p.1009 #29-32)

$$f(x,y) = x + y, R : x^2 + y^2 \le 16, x \ge 0, y \ge 0$$

8. Use a double integral in polar coordinates to find the volume of the solid bounded by the graphs of the equations
(Similar to p.1009 #33-38)

$$z = 3xy, x^2 + y^2 = 4, \text{first octant}$$