

$$1. \quad F(x, y) = xy \vec{i} + y^2 \vec{j}$$

$$a) \quad \vec{r}_1 = \underbrace{(t+1)}_{x(t)} \vec{i} + \underbrace{(6t+3)}_{y(t)} \vec{j} \quad 0 \leq t \leq 1$$

$$F[x(t), y(t)]$$

$$= F\left[\begin{matrix} t+1 \\ x \\ y \end{matrix}, 6t+3\right] = (t+1)(6t+3) \vec{i} + (6t+3)^2 \vec{j} \\ = \underline{(6t^2 + 9t + 3) \vec{i} + (36t^2 + 36t + 9) \vec{j}}$$

$$\vec{r}'_1 = \underline{\vec{i} + 6 \vec{j}}$$

$$F \cdot \vec{r}' = 6t^2 + 9t + 3 + 6(36t^2 + 36t + 9) \\ = 6t^2 + 9t + 3 + 216t^2 + 216t + 54 \\ = 222t^2 + 225t + 57$$

$$\int_0^1 (222t^2 + 225t + 57) dt = \left[\frac{222}{3} t^3 + \frac{225}{2} t^2 + 57t \right]_0^1 \\ = \frac{222}{3} + \frac{225}{2} + 57 = \left(\frac{487}{2} \right)$$

$$b) \quad \vec{r}_2(t) = \underbrace{\left(\frac{1}{3}t + \frac{1}{3}\right)}_{x(t)} \vec{i} + \underbrace{(2t-1)}_{y(t)} \vec{j} \quad 2 \leq t \leq 5$$

$$\vec{F}(x, y) = xy \vec{i} + y^2 \vec{j}$$

$$\vec{F}(x(t), y(t)) = \left(\frac{1}{3}t + \frac{1}{3}\right)(2t-1) \vec{i} + (2t-1)^2 \vec{j} \\ = \underline{\left(\frac{2}{3}t^2 + \frac{1}{3}t - \frac{1}{3}\right) \vec{i} + \underline{(4t^2 - 4t + 1) \vec{j}}}$$

$$\vec{r}'_2 = \underline{\frac{1}{3} \vec{i} + 2 \vec{j}}$$

$$\vec{F} \cdot \vec{r}' = \frac{1}{3} \left(\frac{2}{3}t^2 + \frac{1}{3}t - \frac{1}{3} \right) + 2(4t^2 - 4t + 1) = \frac{2}{9}t^2 + \frac{1}{9}t - \frac{1}{9} + 8t^2 - 8t + 2 \\ = \frac{74}{9}t^2 - \frac{71}{9}t + \frac{17}{9}$$

$$\int_2^5 \left(\frac{74}{9}t^2 - \frac{71}{9}t + \frac{17}{9} \right) dt = 243.5 = \left(\frac{487}{2} \right)$$