Conservative Vector Fields and Independence of Path

1. Show that the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$ is the same for each parametric representation of C
(Similar to p.1090 #1-4)

$$F(x, y) = xyi + y^2 j$$
$$a) r_1(t) = (t+1)i + (6t+3)j, \quad 0 \le t \le 1$$
$$b) r_2(t) = \left(\frac{1}{3}t + \frac{1}{3}\right)i + (2t-1)j, \quad 2 \le t \le 5$$

2. Determine whether the vector field is conservative
(Similar to p.1079 #5-10)

$$F(x, y) = 20x^3y^3i + 15x^4y^2 j$$

$$\text{Hint}: \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Fundamental Theorem of Line Integrals

Let C be a piecewise smooth curve lying in an open region R and given by

r(t) = x(t)**i** + y(t)**j**, a ≤ t ≤ b

If **F**(x, y) = M**i** + N**j** is conservative in R, and M and N are continuous in R, then

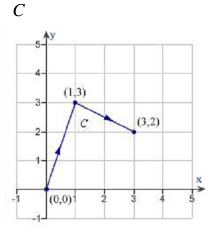
$$\int_C F \cdot dr = \int_C \nabla f \cdot dr = f(x(b), y(b)) - f(x(a), y(a))$$

where f is a potential function of F. That is:

$$F(x, y) = \nabla f(x, y)$$

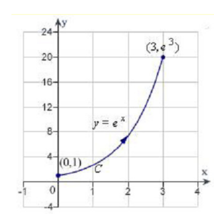
3. Find the value of the line integral (Hint: If F is conservative, the integration may be easier on an alternative path)
(Similar to p.1090 #11-24)

$$\int_C y^5 dx + 5xy^4 dy$$



4. Find the value of the line integral (Hint: If F is conservative, the integration may be easier on an alternative path)
(Similar to p.1090 #11-24)

$$\int_C (3x - y + 4)dx - (x + y - 5)dy$$



5. Find the value of the line integral (Hint: If F is conservative, the integration may be easier on an alternative path)
(Similar to p.1090 #11-24)

$$\int_C (4xy)dx + (2x^2 + 2y^2)dy$$

where C is an ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

from (3, 0) to (0, 2)

Test for Conservative Vector Field in Space

Suppose that M, N, and P have continuous first partial derivatives in an open sphere Q in space. The vector field given by F(x, y, z) = Mi + Nj + Pk is conservative if and only if

Curl F(x, y, z) = 0

That is, F is conservative if and only if:

$$\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}, \frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}, \text{and } \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

6. Find the value of the line integral (Hint: If F is conservative, the integration may be easier on an alternative path)
(Similar to p.1090 #11-24)

$$\int_C F \cdot dr$$

where $F(x, y) = 3i + 2zj + 2yk$

$r(t) = (\cos t)i + (\sin t)j + t^2 k$

$0 \le t \le \pi$

7. Evaluate the line integral using the Fundamental Theorem of Line Integrals
(Similar to p.1090 #25-34)

$$\int_C (7yi + 7xj)dr$$

C : smooth curve from (0, 0) to (2, 4)

8. Find the work done by the force field F in moving an object from P to Q
(Similar to p.1090 #35-36)

F(x, y) = $12x^3y^3$**i** + $(9x^4y^2-1)$**j**, P(0, 0) Q(2, 3)

Independence of Path and Conservative Vector Fields

If **F** is continuous on an open connected region, then the line integral

$$\int_C F \cdot dr$$

is independent of path if and only if **F** is conservative

Equivalent Conditions

Let **F**(x, y, z) = M**i** + N**j** + P**k** have continuous first partial derivatives in an open connected region R, and let C be piecewise smooth curve in R. The following conditions are equivalent.

1) F is conservative. That is $F = \nabla f$ for some function f.

2) $\int_C F \cdot dr$ is independent of path.

3) $\int_C F \cdot dr = 0$ for every closed curve C in R.