13. $\vec{r}(t) = 4t\,\vec{i} + t^3\,\vec{j} + 5t\,\vec{k} \qquad \vec{u}(t) = t^2\,\vec{i} + t\,\vec{j} + 6t\,\vec{k}$

a) $\vec{r}\,'(t) = \boxed{4\vec{i} + 3t^2\vec{j} + 5\vec{k}}$

b) $\vec{r}\,''(t) = \boxed{6t\,\vec{j}}$

c) $\vec{r}(t) \cdot \vec{u}(t) = 4t(t^2) + t^3(t) + (5t)(6t)$

$= 4t^3 + t^4 + 30t^2$

$D_t\left[4t^3 + t^4 + 30t^2\right] = \boxed{12t^2 + 4t^3 + 60t}$

d) $3\vec{r}(t) - \vec{u}(t)$

$= 3\left(4t\vec{i} + t^3\vec{j} + 5t\vec{k}\right) - \left(t^2\vec{i} + t\vec{j} + 6t\vec{k}\right)$

$= 12t\vec{i} + 3t^3\vec{j} + 15t\vec{k} - t^2\vec{i} - t\vec{j} - 6t\vec{k}$

$= (12t - t^2)\vec{i} + (3t^3 - t)\vec{j} + (15t - 6t)\vec{k}$

$= (12t - t^2)\vec{i} + (3t^3 - t)\vec{j} + 9t\vec{k}$

SO DERIV $= \boxed{(12 - 2t)\vec{i} + (9t^2 - 1)\vec{j} + 9\vec{k}}$

e) $\vec{r}(t) \times \vec{u}(t) = \begin{array}{ccc} + & - & + \end{array}$
$$\begin{vmatrix} i & j & k \\ 4t & t^3 & 5t \\ t^2 & t & 6t \end{vmatrix}$$

$= (6t^4 - 5t^2)\vec{i} - (24t^2 - 5t^3)\vec{j} + (4t^2 - t^5)\vec{k}$

DERIV $= \boxed{(24t^3 - 10t)\vec{i} - (48t - 15t^2)\vec{j} + (8t - 5t^4)\vec{k}}$

16
25
41

f) $\|\vec{r}(t)\| = \sqrt{(4t)^2 + (t^3)^2 + (5t)^2} = \sqrt{41t^2 + t^6} = (41t^2 + t^6)^{\frac{1}{2}}$

DERIV $= \frac{1}{2}(41t^2 + t^6)^{-\frac{1}{2}} \cdot (82t + 6t^5) = \boxed{\dfrac{41t + 3t^5}{\sqrt{41t^2 + t^6}}}$