# Differentiation and Integration of Vector-Valued Functions

1. Sketch the plane curve represented by the vector-valued function, and sketch the vectors **r**($t_o$) and **r**'($t_o$) for the given value of $t_o$. Position the vectors such that the initial point of **r**($t_o$) is at the origin and the initial point of **r**'($t_o$) is at the terminal point of **r**($t_o$). What is the relationship between **r**'($t_o$) and the curve?
(Similar to p.848 #1-8)

$$\mathbf{r}(t) = (t-2)\mathbf{i} + t^2\mathbf{j},\ t_o = 1$$

2. Sketch the plane curve represented by the vector-valued function, and sketch the vectors **r**($t_o$) and **r**'($t_o$) for the given value of $t_o$. Position the vectors such that the initial point of **r**($t_o$) is at the origin and the initial point of **r**'($t_o$) is at the terminal point of **r**($t_o$). What is the relationship between **r**'($t_o$) and the curve?
(Similar to p.848 #1-8)

$$\mathbf{r}(t) =< 2\sin(t), 3\cos(t) >,\ t_o = \frac{\pi}{2}$$

3. Sketch the space curve represented by the vector-valued function, and sketch the vectors **r**($t_o$) and **r**'($t_o$) for the given value of $t_o$.
(Similar to p.848 #9-10)

$$\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + 4\mathbf{k},\ t_o = 2$$

4. Find **r**'(t)
(Similar to p.848 #11-22)

$$\mathbf{r}(t) = t^4\mathbf{i} + \sqrt[3]{t}\mathbf{j}$$

5. Find **r**'(t)
(Similar to p.848 #11-22)

$$\mathbf{r}(t) = cos^5 t\mathbf{i} + sin^5 t\mathbf{j} + e^{-t^2}\mathbf{k}$$

6. Find **r**'(t)
(Similar to p.848 #11-22)

$\mathbf{r}(t) =< t^2 \cos(t), \sqrt{t}, 5t >$

7. Find (a) **r**'(t), (b) **r**''(t),
and (c) **r**'(t)·**r**''(t)
(Similar to p.848 #23-30)

$\mathbf{r}(t) = t^4\mathbf{i} - (t^2 - 1)\mathbf{j}$

8. Find (a) **r**'(t), (b) **r**''(t),
and (c) **r**'(t)·**r**''(t)
(Similar to p.848 #23-30)

$\mathbf{r}(t) = < \cos(2t) , \sin(2t) , te^t >$

9. Given the vector-valued function,
find the unit vectors:
$\mathbf{r}'(t_o)/\|\mathbf{r}'(t_o)\|$ and $\mathbf{r}''(t_o)/\|\mathbf{r}''(t_o)\|$
(Similar to p.848 #31-32)

$\mathbf{r}(t) = \cos(t)\,\mathbf{i} + \sin(t)\,\mathbf{j} + t^3\mathbf{k}$,
$t_o = \pi/2$

## Smooth Curve

Given a vector-valued function:

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

is smooth on an open interval I if f', g', and h' are continuous on I and $\mathbf{r}'(t) \neq \mathbf{0}$ for any value of t in the interval I

10. Find the open interval(s) on which the curve given by the vector-valued function is smooth
(Similar to p.848 #33-42)

$\mathbf{r}(t) = t^4\mathbf{i} + 3t^2\mathbf{j}$

11. Find the open interval(s) on which the curve given by the vector-valued function is smooth
(Similar to p.848 #33-42)

$$\mathbf{r}(t) = cos^5(t)\mathbf{i} + sin^3(t)\mathbf{j}$$

12. Find the open interval(s) on which the curve given by the vector-valued function is smooth
(Similar to p.848 #33-42)

$$\mathbf{r}(t) = \frac{1}{t-2}\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$$

13. Use the properties of the derivative to find the following
(a) r'(t) (b) r''(t) (c) $D_t$[r(t)·u(t)]
(d) $D_t$[3r(t) – u(t)] (e) $D_t$[r(t) x u(t)]
(f) $D_t$[||r(t)||], t > 0
(Similar to p.848 #43-44)

$$\mathbf{r}(t) = 4t\mathbf{i} + t^3\mathbf{j} + 5t\mathbf{k}$$
$$\mathbf{u}(t) = t^2\mathbf{i} + \mathrm{t}\mathbf{j} + 6t\mathbf{k}$$

14. Use the definition of the derivative to find r'(t)
(Similar to p.849 #49-51)

$$\mathbf{r}(t) = (5t-1)\mathbf{i} + t^2\mathbf{j}$$

15. Find the indefinite integral
(Similar to p.849 #53-60)

$$\int (3t\mathbf{i} + \mathbf{j} - 2\mathbf{k})dt$$

16. Find the indefinite integral
(Similar to p.849 #53-60)

$$\int \left(\sqrt[3]{t}\mathbf{i} + (\sin \mathrm{t})\mathbf{j} - e^{2t-1}\mathbf{k}\right)dt$$

17. Evaluate the definite integral
(Similar to p.849 #61-66)

$$\int_0^4 \left(t^2\mathbf{i} + \sqrt{t}\mathbf{j} - t\mathbf{k}\right)dt$$

18. Find **r**(t) for the given conditions
(Similar to p.849 #67-72)

$\boldsymbol{r}'(t) = 9e^{3t}\mathbf{i} - t\mathbf{j}$, **r**(0) = 2**j**

19. Find **r**(t) for the given conditions
(Similar to p.849 #67-72)

$\boldsymbol{r}'(t) = te^{t}\mathbf{i} - t^2\mathbf{j} + 3\mathbf{k}$,
**r**(0) = 4**i** + 2**j** - **k**