3. $f(x,y,z) = x^2yz^3 \quad P(2,1,-4) \quad \vec{v} = \langle 2,3,1 \rangle$

① $\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{\langle 2,3,1\rangle}{\sqrt{2^2+3^2+1^2}} = \dfrac{\langle 2,3,1\rangle}{\sqrt{4+9+1}} = \dfrac{\langle 2,3,1\rangle}{\sqrt{14}} = \left\langle \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}, \dfrac{1}{\sqrt{14}} \right\rangle$

$$= \left\langle \frac{2\sqrt{14}}{14}, \frac{3\sqrt{14}}{14}, \frac{\sqrt{14}}{14} \right\rangle$$

$$\vec{u} = \left\langle \frac{\sqrt{14}}{7}, \frac{3\sqrt{14}}{14}, \frac{\sqrt{14}}{14} \right\rangle$$

② $\nabla f(x,y,z) = f_x \vec{i} + f_y \vec{j} + f_z \vec{k}$

$$= 2xyz^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2\vec{k}$$

③ $\nabla f(\underset{x}{2}, \underset{y}{1}, \underset{z}{-4}) = 2(2)(1)(-4)^3\vec{i} + (2)^2(-4)^3\vec{j} + 3(2)^2(1)(-4)^2\vec{k}$

$$= 4(-64)\vec{i} + 4(-64)\vec{j} + 12(16)\vec{k}$$

$$= -256\vec{i} - 256\vec{j} + 192\vec{k}$$

④ $D_u f(2,1,-4) = \nabla f(2,1,-4) \cdot \vec{u}$

$$= (-256\vec{i} - 256\vec{j} + 192\vec{k}) \cdot \left( \left\langle \frac{\sqrt{14}}{7}, \frac{3\sqrt{14}}{14}, \frac{\sqrt{14}}{14} \right\rangle \right)$$

$$= \frac{-256\sqrt{14}}{7} + \frac{-256 \cdot 3\sqrt{14}}{14} + \frac{192\sqrt{14}}{14}$$

$$= \frac{-512\sqrt{14} - 768\sqrt{14} + 192\sqrt{14}}{14}$$

$$= \frac{-1088\sqrt{14}}{14}$$

$$= \boxed{\frac{-544\sqrt{14}}{7}}$$