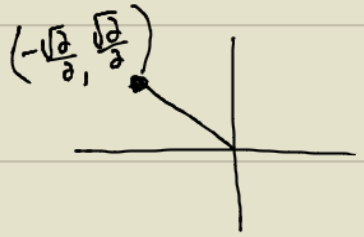
4. $f(x,y) = x^2 + y^3$, $\theta = \frac{3\pi}{4}$



① $\vec{u} = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$

$\vec{u} = \cos\frac{3\pi}{4}\vec{i} + \sin\frac{3\pi}{4}\vec{j}$

$\vec{u} = -\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}$

② $\nabla f(x,y) = f_x\vec{i} + f_y\vec{j}$

$= 2x\vec{i} + 3y^2\vec{j}$

③ ④ $D_u f(x,y) = \nabla f(x,y) \cdot \vec{u}$

$= (2x\vec{i} + 3y^2\vec{j}) \cdot (-\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j})$

$= 2x(-\frac{\sqrt{2}}{2}) + 3y^2(\frac{\sqrt{2}}{2})$

$= \boxed{-\sqrt{2}x + \frac{3\sqrt{2}}{2}y^2}$