

$$5. f(x,y) = xe^y \quad P(2,1) \quad Q(5,3)$$

$$\vec{v} = \langle 5-2, 3-1 \rangle = \langle 3, 2 \rangle$$

$$\textcircled{1} \vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 3, 2 \rangle}{\sqrt{3^2 + 2^2}} = \frac{\langle 3, 2 \rangle}{\sqrt{13}} = \left\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle = \left\langle \frac{3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13} \right\rangle$$

$$\textcircled{2} \nabla f(x,y) = f_x \vec{i} + f_y \vec{j} \\ = e^y \vec{i} + xe^y \vec{j}$$

$$\textcircled{3} \nabla f \Big|_{\substack{x=2 \\ y=1}} = e^1 \vec{i} + 2e^1 \vec{j} \\ = e \vec{i} + 2e \vec{j}$$

$$\textcircled{4} D_u f(2,1) = \nabla f(2,1) \cdot \vec{u} \\ = (e \vec{i} + 2e \vec{j}) \cdot \left\langle \frac{3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13} \right\rangle \\ = \frac{3e\sqrt{13}}{13} + \frac{4e\sqrt{13}}{13} \\ = \frac{7e\sqrt{13}}{13}$$