

$$8. f(x,y) = 5x - y^4 - 3 \quad P(2,0) \quad Q(5,1)$$

$$\vec{V} = \langle 5-2, 1-0 \rangle = \langle 3, 1 \rangle$$

$$\textcircled{1} \vec{u} = \frac{\vec{V}}{\|\vec{V}\|} = \frac{\langle 3, 1 \rangle}{\sqrt{3^2+1^2}} = \frac{\langle 3, 1 \rangle}{\sqrt{10}} = \left\langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle = \left\langle \frac{3\sqrt{10}}{10}, \frac{\sqrt{10}}{10} \right\rangle$$

$$\textcircled{2} \nabla f(x,y) = f_x \vec{i} + f_y \vec{j} \\ = 5 \vec{i} + (-4y^3) \vec{j}$$

$$\textcircled{3} \nabla f \Big|_{\substack{x=2 \\ y=0}} = 5 \vec{i} - 4(0)^3 \vec{j} \\ = 5 \vec{i}$$

$$\textcircled{4} D_u f(2,0) = \nabla f(2,0) \cdot \vec{u} \\ = (5 \vec{i}) \cdot \left\langle \frac{3\sqrt{10}}{10}, \frac{\sqrt{10}}{10} \right\rangle \\ = \frac{15\sqrt{10}}{10} \\ = \frac{3\sqrt{10}}{2}$$