Directional Derivatives and Gradients

Finding Directional Derivatives at a point (x_o, y_o) in the direction of a vector **v**

1. Find the unit vector (**u**) of the given vector: $\boldsymbol{u} = \frac{\boldsymbol{v}}{\|v\|}$
2. Find $\nabla f(x, y) = f_x(x, y)\boldsymbol{i} + f_y(x, y)\mathbf{j}$
3. Find $\nabla f(x_o, yo)$
4. $D_u f(x_o, y_o) = \nabla f(x_o, yo) \cdot \mathbf{u}$

1. Find the directional derivative of the function at P in the direction of **v**
(Similar to p.942 #1-12)

$$f(x, y) = 5x + 2xy - 3y, \quad P(3,5), v = \frac{4}{5}i + \frac{3}{5}j$$

2. Find the directional derivative of the function at P in the direction of **v**
(Similar to p.942 #1-12)

$$f(x, y) = e^{2x} - \cos y, \quad P(0, \pi), v = 2i + 4j$$

3. Find the directional derivative of the function at P in the direction of **v**
(Similar to p.942 #1-12)

$$f(x, y, z) = x^2 yz^3, \quad P(2,1,-4), v = < 2,3,1 >$$

4. Find the directional derivative of the function in the direction of the unit vector **u** = cos(θ)**i** + sin(θ)**j**
(Similar to p.942 #13-16)

$$f(x, y) = x^2 + y^3, \quad \theta = \frac{3\pi}{4}$$

5. Find the directional derivative of the function at P in the direction of Q
(Similar to p.942 #17-20)

$$f(x, y) = xe^{y}, \quad P(2,1), Q(5,3)$$

6. Find the gradient of the function at the given point
(Similar to p.942 #21-26)

$$f(x, y) = 5x^2 - y^3, \quad (3,1)$$

$$\nabla f(x, y) = f_x(x, y)\mathrm{i} + f_y(x, y)\mathrm{j}$$

7. Find the gradient of the function at the given point
(Similar to p.942 #21-26)

$$f(x, y) = \sin(x^3 - y), \quad (1,2)$$

8. Use the gradient to find the directional derivative of the function at P in the direction of Q
(Similar to p.942 #27-30)

$$f(x, y) = 5x - y^4 - 3, \quad P(2,0), Q(5,1)$$

9. Find the gradient of the function and the maximum value of the directional derivative at the given point
(Similar to p.942 #31-40)

$$f(x, y) = \frac{x^2 - y}{2y + 3}, \quad (1,2)$$

$$\text{maximum value}: \left\|\nabla f(x, y)\right\|$$

$$\text{minimum value}: -\left\|\nabla f(x, y)\right\|$$

10. Find the gradient of the function and the maximum value of the directional derivative at the given point
(Similar to p.942 #31-40)

$$f(x, y, z) = \sqrt{x^2 - y^2 + 3z^2}, \quad (1,0,2)$$