3. $\vec{F}(x,y,z) = (2x)\vec{i} + (y^2)\vec{j} + (2z)\vec{k}$

S: $x^2+y^2=9$, $z=0$, $z=5$

$y = r\sin\theta$

$x = r\cos\theta$

$$\operatorname{div}\vec{F} = 2 + 2y + 0 = 2y + 2$$

$$\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=3}\int_{z=0}^{z=5} (2\cdot r\sin\theta + 2)\, r \, dz\, dr\, d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=3}\int_{z=0}^{z=5} (2r^2\sin\theta + 2r)\, dz\, dr\, d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=3} \left[2r^2\sin\theta\, z + 2rz\right]_{z=0}^{z=5} dr\, d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=3} \left[2r^2\sin\theta\cdot 5 + 2r\cdot 5\right] dr\, d\theta$$

$$= 2\cdot 5 \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=3} \left[r^2\sin\theta + 2r\right] dr\, d\theta$$

$$= 10 \int_{\theta=0}^{\theta=2\pi} \left[\tfrac{1}{3}r^3\sin\theta + 2\cdot\tfrac{1}{2}r^2\right]_{r=0}^{r=3} d\theta$$

$$= 10 \int_{\theta=0}^{\theta=2\pi} \left[9\sin\theta + 9\right] d\theta$$

$$= 10 \left[-9\cos\theta + 9\theta\right]_{\theta=0}^{\theta=2\pi}$$

$$= 10\left[-9\cos 2\pi + 9(2\pi) - \left(-9\cos 0 + 9(0)\right)\right]$$

$$= 10\left[-9 + 18\pi - (-9)\right]$$

$$= 10\,(18\pi)$$

$$= 180\pi$$