

$$7. \quad x^2 + z^2 = 4 \quad y^2 + z^2 = 4$$

$$\begin{aligned} x^2 + z^2 &= 4 \\ -y^2 + z^2 &= 4 \\ x^2 - y^2 &= 0 \\ x^2 &= y^2 \\ x &= y \end{aligned}$$

$$\begin{aligned} x^2 + z^2 &= 4 \\ z^2 &= 4 - x^2 \\ z &= \sqrt{4 - x^2} \end{aligned}$$

$$= 2 \int_{x=0}^{x=2} \int_{y=0}^{y=x} \sqrt{4-x^2} \, dy \, dx$$

$$= 2 \int_{x=0}^{x=2} \left[y \sqrt{4-x^2} \right]_{y=0}^{y=x} dx$$

$$= 2 \int_{x=0}^{x=2} \left[x \sqrt{4-x^2} - 0 \sqrt{4-x^2} \right] dx$$

$$= 2 \int_{x=0}^{x=2} x (4-x^2)^{\frac{1}{2}} dx$$

$$u = 4 - x^2 \quad du = -2x \, dx$$

$$= 2 \cdot -\frac{1}{2} \int_{x=0}^{x=2} -2x (4-x^2)^{\frac{1}{2}} dx$$

$$= -1 \int_{x=0}^{x=2} u^{\frac{1}{2}} du$$

$$= -1 \left[\frac{2}{3} u^{\frac{3}{2}} \right]_{x=0}^{x=2}$$

$$= -\frac{2}{3} \left[(4-x^2)^{\frac{3}{2}} \right]_{x=0}^{x=2}$$

$$= -\frac{2}{3} \left[(4-2^2)^{\frac{3}{2}} - (4-0^2)^{\frac{3}{2}} \right]$$

$$= -\frac{2}{3} (-4^{\frac{3}{2}})$$

$$\begin{aligned} &= \frac{2}{3} \cdot (4^{\frac{3}{2}}) \\ &= \frac{2}{3} (4^{\frac{3}{2}})^3 \\ &= \frac{2}{3} (\sqrt{4})^3 \\ &= \frac{2}{3} \cdot 2^3 \\ &= \frac{16}{3} \end{aligned}$$