

$$1. f(x, y) = (x+4)^2 + (y-2)^2$$

$$f_x = \partial(x+4)^2 \cdot 1 \quad f_y = \partial(y-2)^2 \cdot 1 \quad f_{xx} = 2 \quad f_{yy} = 2 \quad f_{xy} = 0$$

$$= 2x+8 \quad = 2y-4$$

$$2x+8=0$$

$$2x = -8$$

$$x = -4$$

$$2y-4=0$$

$$2y = 4$$

$$y = 2$$

$$d = f_{xx} f_{yy} - f_{xy}^2$$

$$d = 2(2) - 0^2$$

$$d = 4$$

REL. MIN $(-4, 2, f(-4, 2))$

REL. MIN $(-4, 2, 0)$

$$2. f(x, y) = x^2 + y^2 + 4x - 8y + 2$$

$$f_x = 2x + 4 \quad f_y = 2y - 8 \quad f_{xx} = 2 \quad f_{yy} = 2 \quad f_{xy} = 0$$

$$2x+4=0$$

$$2x = -4$$

$$x = -2$$

$$2y-8=0$$

$$2y = 8$$

$$y = 4$$

$$d = f_{xx} f_{yy} - f_{xy}^2$$

$$d = 2(2) - 0^2$$

$$d = 4$$

$(-2, 4, f(-2, 4))$

REL. MIN $(-2, 4, -18)$

$$f(-2, 4) = (-2)^2 + 4^2 + 4(-2) - 8(4) + 2$$

$$= 4 + 16 - 8 - 32 + 2$$

$$= 18 - 30$$

$$= -18$$

$$3. f(x, y) = -x^2 - 3y^2 + 8x - 12y + 3$$

$$f_x = -2x + 8 \quad f_y = -6y - 12 \quad f_{xx} = -2 \quad f_{yy} = -6 \quad f_{xy} = 0$$

$$-2x+8=0$$

$$8 = 2x$$

$$x = 4$$

$$-6y-12=0$$

$$-12 = 6y$$

$$y = -2$$

$$d = f_{xx} f_{yy} - f_{xy}^2$$

$$d = (-2)(-6) - 0^2$$

$$d = 12$$

REL. MAX $(4, -2, f(4, -2))$

REL. MAX $(4, -2, 31)$

$$f(4, -2) = -(4)^2 - 3(-2)^2 + 8(4) - 12(-2) + 3$$

$$= -16 - 12 + 32 + 24 + 3$$

$$= -28 + 29$$

$$= 31$$