Extrema of Functions of Two Variables

Second Partials Test

Let f have continuous second partial derivatives on an open region containing a point (a, b) for which $f_x(a, b) = 0$ and $f_y(a, b) = 0$

Let $d = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$

1. If $d > 0$ and $f_{xx}(a, b) > 0$,
   relative min at (a, b, f(a,b))
2. If $d > 0$ and $f_{xx}(a, b) < 0$,
   relative max at (a, b, f(a, b))
3. If $d < 0$, then (a, b, f(a, b)) is a saddle point
4. The test is inconclusive if $d = 0$

1. Find any critical points and test for relative extrema.
(Similar to p.960 #1-6)

$$f(x, y) = (x+4)^2 + (y-2)^2$$

2. Find any critical points and test for relative extrema.
(Similar to p.960 #1-6)

$$f(x, y) = x^2 + y^2 + 4x - 8y + 2$$

3. Find any critical points and test for relative extrema.
(Similar to p.960 #7-16)

$$f(x, y) = -x^2 - 3y^2 + 8x - 12y + 3$$

4. Find any critical points and test for relative extrema.
(Similar to p.960 #7-16)

$$f(x, y) = \sqrt{3x^2 + y^2}$$

5. Find any critical points and test for relative extrema.
(Similar to p.960 #7-16)

$$f(x, y) = 2 - |x+3| - |y-1|$$

6. Examine the function for relative extrema and saddle points
(Similar to p.960 #21-28)

$$f(x, y) = xy - 2x$$

7. Examine the function for relative extrema and saddle points
(Similar to p.960 #21-28)

$$f(x, y) = e^x \cos y$$

8. Find the critical points of the function and, from the form of the function, determine whether a relative maximum or a relative minimum occurs at each point
(Similar to p.961 #43-44)

$$f(x, y, z) = (x-2)^2 + (y+1)^2 + (z-5)^2$$

Absolute Extrema

- Find the first partials, set them equal to zero and find the critical points (make sure they are in the region)
- Find the high and low points of the boundaries (plug in either x or y and then take derivative)
- Find the intersections
- Plug all of the above into the function, largest value is your absolute maximum point, smallest value is your absolute minimum point

9. Find the absolute extrema of the function over the region R. (In each case, R contains the boundaries.) Use a computer algebra system to confirm your results
(Similar to p.961 #45-54)

$$f(x, y) = x^2 + xy$$
$$R = \{(x, y) : -3 \le x \le 3, -1 \le y \le 1\}$$