

$$6. f(x, y) = 3x^2 + y \quad (z = 3x^2 + y)$$

$$a) \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

$$\begin{aligned} \text{1st: } f(\underbrace{x + \Delta x}_x, \underbrace{y}_y) &= 3(x + \Delta x)^2 + (y) \\ &= 3(x + \Delta x)(x + \Delta x) + y \\ &= 3(x^2 + x\Delta x + x\Delta x + (\Delta x)^2) + y \\ &= 3(x^2 + 2x\Delta x + (\Delta x)^2) + y \\ &= 3x^2 + 6x\Delta x + 3(\Delta x)^2 + y \end{aligned}$$

$$\text{2nd: } \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

$$= \frac{3x^2 + 6x\Delta x + 3(\Delta x)^2 + y - (3x^2 + y)}{\Delta x}$$

$$= \frac{3x^2 + 6x\Delta x + 3(\Delta x)^2 + y - 3x^2 - y}{\Delta x}$$

$$= \frac{6x\Delta x + 3(\Delta x)^2}{\Delta x}$$

$$= \frac{\cancel{\Delta x}(6x + 3\Delta x)}{\cancel{\Delta x}}$$

$$= (6x + 3\Delta x)$$