Green's Theorem

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Let R be a simply connected region with a piecewise smooth boundary C, oriented counterclockwise (that is, C is traversed once so that the region R always lies to the left). If M and N have continuous first partial derivatives in an open region containing R, then

$$\int_C M\ dx + N\ dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

1. Use Green's Theorem to evaluate the integral (Similar to p.1099 #7-10)

$$\int_C (y-x)dx + (8x-y)dy$$

for the path C: boundary of the region lying between the graphs of $y = x$ and $y = x^2 - 8x$

2. Use Green's Theorem to evaluate the integral (Similar to p.1099 #7-10)

$$\int_C (y-x)dx + (2x-y)dy$$

for the path C defined as $x = 5\cos\theta,\ y = 3\sin\theta$

Hint: Area of Ellipse $= \pi ab$

3. Use Green's Theorem to evaluate the integral (Similar to p.1099 #7-10)

$$\int_C (y-x)dx + (2x-y)dy$$

where C is the boundary of the region lying inside the rectangle bounded by $x = -2, x = 2, y = -4, y = 4$, and outside the square bounded by $x = -1, x = 1$, $y = -1$, and $y = 1$

4. Use Green's Theorem to evaluate the integral (Similar to p.1099 #11-20)

$$\int_C (3xy)dx + (x+y)dy$$

C: boundary of the region lying between the graphs of $y = 0$ and $y = 4 - x^2$

5. Use Green's Theorem to evaluate the integral
(Similar to p.1099 #11-20)

$$\int_C (y^2)dx + (3xy)dy$$

C : boundary of the region lying between the graphs of $y = 0$, $y = \sqrt{x}$ and $x = 16$

6. Use Green's Theorem to evaluate the integral
(Similar to p.1099 #11-20)

$$\int_C (x^2 - y^2)dx + (10xy)dy$$

for the path $C: x^2 + y^2 = 9$

7. Use Green's Theorem to calculate the work done by the force F on a particle that is moving counterclockwise around the closed path C
(Similar to p.1099 #21-24)

$$F(x, y) = 5xyi + (x + y)j$$

$$C: x^2 + y^2 = 16$$

Hint : Change to polar form