15. $\int_{x=0}^{x=4}\int_{y=x}^{y=4} x\sqrt[5]{2+y^3}\,dy\,dx$

$= \int_{y=0}^{y=4}\int_{x=0}^{x=y} x(2+y^3)^{\frac{1}{5}}\,dx\,dy$

$= \int_{y=0}^{y=4} \left[\frac{1}{2}x^2(2+y^3)^{\frac{1}{5}}\right]_{x=0}^{x=y} dy$

$= \frac{1}{2}\int_{y=0}^{y=4} \left[y^2(2+y^3)^{\frac{1}{5}} - 0^2(2+y^3)^{\frac{1}{5}}\right] dy$

$= \frac{1}{2}\int_{y=0}^{y=4} \left[y^2(2+y^3)^{\frac{1}{5}}\right] dy$

$u = 2+y^3 \quad du = 3y^2\,dy$

$= \frac{1}{2}\cdot\frac{1}{3}\int_{y=0}^{y=4} 3y^2(2+y^3)^{\frac{1}{5}}\,dy$

$= \frac{1}{6}\int_{y=0}^{y=4} u^{\frac{1}{5}}\,du$

$= \frac{1}{6}\left[\frac{u^{\frac{1}{5}+1}}{\frac{1}{5}+1}\right]_{y=0}^{y=4}$

$= \frac{1}{6}\left[\frac{5}{6}u^{\frac{6}{5}}\right]_{y=0}^{y=4}$

$= \frac{5}{36}\left[(2+y^3)^{\frac{6}{5}}\right]_{y=0}^{y=4}$

$= \frac{5}{36}\left[(2+4^3)^{\frac{6}{5}} - (2+0^3)^{\frac{6}{5}}\right]$

$= \frac{5}{36}\left(66^{\frac{6}{5}} - 2^{\frac{6}{5}}\right)$

$= \frac{5}{36}\left(\sqrt[5]{66^6} - \sqrt[5]{2^6}\right)$

$= \frac{5}{36}\left(66\sqrt[5]{66} - 2\sqrt[5]{2}\right)$

$= \frac{5}{18}\left(33\sqrt[5]{66} - \sqrt[5]{2}\right)$