

$$1. f(x, y) = x^2 + 3y^2$$

$$\textcircled{1} \frac{x+4y-7}{9} = 0$$

$$\textcircled{2} \nabla f = f_x \vec{i} + f_y \vec{j}$$
$$\nabla f = 2x \vec{i} + 6y \vec{j}$$

$$\nabla g = g_x \vec{i} + g_y \vec{j}$$
$$\nabla g = \vec{i} + 4 \vec{j}$$

$$\textcircled{3} \nabla f = \lambda \nabla g$$

$$2x \vec{i} + 6y \vec{j} = \lambda (\vec{i} + 4 \vec{j})$$

$$2x \vec{i} + 6y \vec{j} = \lambda \vec{i} + 4\lambda \vec{j}$$

$$2x = \lambda \quad 6y = 4\lambda$$

$$x = \frac{1}{2} \lambda \quad y = \frac{4}{6} \lambda$$

$$y = \frac{2}{3} \lambda$$

$$x + 4y - 7 = 0$$

$$\frac{1}{2} \lambda + 4 \left(\frac{2}{3} \lambda \right) - 7 = 0$$

$$\frac{1}{2} \lambda + \frac{8}{3} \lambda - 7 = 0$$

$$6 \left(\frac{1}{2} \lambda \right) + 6 \left(\frac{8}{3} \lambda \right) + 6(-7) = 6(0)$$

$$3\lambda + 16\lambda - 42 = 0$$

$$19\lambda = 42$$

$$\lambda = \frac{42}{19}$$

$$x = \frac{1}{2} \lambda \quad y = \frac{2}{3} \lambda$$

$$x = \frac{1}{2} \left(\frac{42}{19} \right) \quad y = \frac{2}{3} \left(\frac{42}{19} \right)$$

$$x = \frac{21}{19} \quad y = \frac{28}{19}$$

$$f(x, y) = x^2 + 3y^2$$

$$f\left(\frac{21}{19}, \frac{28}{19}\right) = \left(\frac{21}{19}\right)^2 + 3 \left(\frac{28}{19}\right)^2$$

$$= \frac{147}{19}$$