2. $f(x,y) = \sqrt{x^2 - y^2}$

$f(x,y) = (x^2 - y^2)^{1/2}$

$3x + 2y = 5$

① $3x + 2y - 5 = 0$

g

② $\nabla f = f_x \vec{i} + f_y \vec{j}$

$\nabla f = \frac{1}{2}(x^2 - y^2)^{-\frac{1}{2}} \cdot 2x \vec{i} + \frac{1}{2}(x^2 - y^2)^{-\frac{1}{2}} \cdot (-2y) \vec{j}$

$= \frac{x}{(x^2 - y^2)^{1/2}} \vec{i} - \frac{y}{(x^2 - y^2)^{1/2}} \vec{j}$

$\nabla g = g_x \vec{i} + g_y \vec{j}$

$\nabla g = 3\vec{i} + 2\vec{j}$

③ $\nabla f = \lambda \nabla g$

$\frac{x}{\sqrt{x^2 - y^2}} \vec{i} - \frac{y}{\sqrt{x^2 - y^2}} \vec{j} = \lambda (3\vec{i} + 2\vec{j})$

$\frac{x}{\sqrt{x^2 - y^2}} = 3\lambda \qquad \frac{-y}{\sqrt{x^2 - y^2}} = 2\lambda$

$\frac{\frac{1}{3}x}{\sqrt{x^2 - y^2}} = \lambda \qquad \frac{-\frac{1}{2}y}{\sqrt{x^2 - y^2}} = \lambda$

$\frac{\frac{1}{3}x}{\sqrt{x^2 - y^2}} = \frac{-\frac{1}{2}y}{\sqrt{x^2 - y^2}}$

$\frac{1}{3}x = -\frac{1}{2}y$

$3\left(\frac{1}{3}x\right) = 3\left(-\frac{1}{2}y\right)$

$x = -\frac{3}{2}y$

$3x + 2y = 5$

$3\left(-\frac{3}{2}y\right) + 2y = 5$

$-\frac{9}{2}y + 2y = 5$

$-9y + 4y = 10$

$-5y = 10$

$y = -2$

so

$x = -\frac{3}{2}y$

$x = -\frac{3}{2}(-2)$

$x = 3 \quad y = -2$

$f(x,y) = \sqrt{x^2 - y^2}$

$f(3,-2) = \sqrt{3^2 - (-2)^2}$

$= \sqrt{9 - 4}$

$= \sqrt{5}$