

$$3. f(x, y, z) = x^2 + y^2 + z^2 \quad \textcircled{1} \quad \frac{x+y+z-5=0}{9}$$

$$\textcircled{2} \quad \begin{aligned} \nabla f &= f_x \vec{i} + f_y \vec{j} + f_z \vec{k} & \nabla g &= g_x \vec{i} + g_y \vec{j} + g_z \vec{k} \\ \nabla f &= 2x \vec{i} + 2y \vec{j} + 2z \vec{k} & \nabla g &= \vec{i} + \vec{j} + \vec{k} \end{aligned}$$

$$\textcircled{3} \quad \begin{aligned} \nabla f &= \lambda \nabla g \\ 2x \vec{i} + 2y \vec{j} + 2z \vec{k} &= \lambda (\vec{i} + \vec{j} + \vec{k}) \\ 2x \vec{i} + 2y \vec{j} + 2z \vec{k} &= \lambda \vec{i} + \lambda \vec{j} + \lambda \vec{k} \\ 2x = \lambda \quad 2y = \lambda \quad 2z = \lambda \end{aligned}$$

$$\begin{aligned} 2x &= 2y = 2z \\ x &= y = z \end{aligned}$$

$$x + y + z - 5 = 0$$

$$x + x + x - 5 = 0$$

$$3x = 5$$

$$\Rightarrow \left(x = \frac{5}{3}, y = \frac{5}{3}, z = \frac{5}{3} \right)$$

$$f(x, y, z) = x^2 + y^2 + z^2$$

$$f\left(\frac{5}{3}, \frac{5}{3}, \frac{5}{3}\right) = \left(\frac{5}{3}\right)^2 + \left(\frac{5}{3}\right)^2 + \left(\frac{5}{3}\right)^2$$

$$= \frac{25}{9} + \frac{25}{9} + \frac{25}{9}$$

$$= \left(\frac{75}{9} \right)$$