3. $f(x,y,z) = x^2 + y^2 + z^2$ $\quad$ ① $\underbrace{x+y+z-5=0}_{g}$

②

$\nabla f = f_x\vec{i} + f_y\vec{j} + f_z\vec{k}$ $\qquad$ $\nabla g = g_x\vec{i} + g_y\vec{j} + g_z\vec{k}$

$\nabla f = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$ $\qquad$ $\nabla g = \vec{i} + \vec{j} + \vec{k}$

③

$\nabla f = \lambda \nabla g$

$2x\vec{i} + 2y\vec{j} + 2z\vec{k} = \lambda(\vec{i} + \vec{j} + \vec{k})$

$2x\vec{i} + 2y\vec{j} + 2z\vec{k} = \lambda\vec{i} + \lambda\vec{j} + \lambda\vec{k}$

$2x = \lambda \quad 2y = \lambda \quad 2z = \lambda$

$\lambda = 2x \quad \lambda = 2y \quad \lambda = 2z$

$2x = 2y = 2z$

$x = y = z$

$x + y + z - 5 = 0$

$x + x + x - 5 = 0$

$3x = 5$

$x = \frac{5}{3}$

$\Rightarrow$ $y = \frac{5}{3}, \; z = \frac{5}{3}$

$f(x,y,z) = x^2 + y^2 + z^2$

$f\left(\frac{5}{3}, \frac{5}{3}, \frac{5}{3}\right) = \left(\frac{5}{3}\right)^2 + \left(\frac{5}{3}\right)^2 + \left(\frac{5}{3}\right)^2$

$= \frac{25}{9} + \frac{25}{9} + \frac{25}{9}$

$= \frac{75}{9}$