4. $f(x,y,z) = xyz$ $\quad x+y+z=20$ $\quad \underbrace{x-y+z}_{h}=0$

① $\underbrace{x+y+z-20}_{g}=0$

②

$\nabla f = f_x\vec{i} + f_y\vec{j} + f_z\vec{k}$ $\quad \nabla g = g_x\vec{i} + g_y\vec{j} + g_z\vec{k}$ $\quad \nabla h = h_x\vec{i} + h_y\vec{j} + h_z\vec{k}$

$\nabla f = yz\vec{i} + xz\vec{j} + xy\vec{k}$ $\quad \nabla g = \vec{i} + \vec{j} + \vec{k}$ $\quad \nabla h = \vec{i} - \vec{j} + \vec{k}$

③ $\nabla f = \lambda \nabla g + \mu \nabla h$

$yz\vec{i} + xz\vec{j} + xy\vec{k} = \lambda(\vec{i} + \vec{j} + \vec{k}) + \mu(\vec{i} - \vec{j} + \vec{k})$

$yz\vec{i} + xz\vec{j} + xy\vec{k} = \lambda\vec{i} + \lambda\vec{j} + \lambda\vec{k} + \mu\vec{i} - \mu\vec{j} + \mu\vec{k}$

$yz\vec{i} + xz\vec{j} + xy\vec{k} = (\lambda+\mu)\vec{i} + (\lambda-\mu)\vec{j} + (\lambda+\mu)\vec{k}$

$\underline{yz = \lambda+\mu}$ $\quad xz = \lambda-\mu$ $\quad \underline{xy = \lambda+\mu}$

$yz = xy$

$z = x$

$\begin{cases} x+y+z=20 \\ x-y+z=0 \end{cases}$

$\begin{cases} x+y+x=20 \\ x-y+x=0 \end{cases}$

$\begin{cases} 2x+y=20 \\ \underline{2x-\cancel{y}=\underline{0}} \end{cases}$

$4x = 20$

$x=5$

So $z=5$

AND $2x-y=0$

$2(5)-y=0$

$10=y$

$x=5,\ y=10,\ z=5$

$f(x,y,z) = xyz$

$f(5,10,5) = 5(10)(5)$

$= 250$