

## Lagrange Multipliers

### Lagrange Multipliers with One Constraint

1. Get zero on one side of the constraint, the other side is  $g$
2. Find  $\nabla f$  and  $\nabla g$
3. Solve  $\nabla f = \lambda \nabla g$ 
  - Either:
    - a) Find  $x = \text{something}\lambda$ ,  $y = \text{something}\lambda$
    - b) Plug these into constraint and solve for  $\lambda$
    - c) Now plug  $\lambda$  into  $x = \text{something}\lambda$ ,  
 $y = \text{something}\lambda$  and find  $x, y$
  - Or:
    - a) solve both equations for  $\lambda$  and equate them

1. Use Lagrange multipliers to find the indicated extrema, assuming that  $x$  and  $y$  are positive  
(Similar to p.976 #5-10)

$$\text{Minimize } f(x, y) = x^2 + 3y^2$$

$$\text{Constraint : } x + 4y - 7 = 0$$

2. Use Lagrange multipliers to find the indicated extrema, assuming that  $x$  and  $y$  are positive  
(Similar to p.976 #5-10)

$$\text{Minimize } f(x, y) = \sqrt{x^2 - y^2}$$

$$\text{Constraint : } 3x + 2y = 5$$

3. Use Lagrange multipliers to find the indicated extrema, assuming that  $x, y$  and  $z$  are positive  
(Similar to p.976 #11-14)

$$\text{Minimize } f(x, y, z) = x^2 + y^2 + z^2$$

$$\text{Constraint : } x + y + z - 5 = 0$$

4. Use Lagrange multipliers to find the indicated extrema of  $f$  subject to two constraints. In each case, assume that  $x, y$  and  $z$  are nonnegative  
(Similar to p.976 #17-18)

$$\text{Maximize } f(x, y, z) = xyz$$

$$\text{Constraint : } x + y + z = 20, x - y + z = 0$$

$$\nabla f = \lambda \nabla g + \mu \nabla h$$

5. Use Lagrange multipliers to find the minimum distance from the curve or surface to the indicated point. [Hint: use  $\min f(x, y) = (x-x_0)^2 + (y-y_0)^2$  (Similar to p.976 #19-25)]

*Curve:* *Line:*  $4x - y = 2$

*Point:* (3,5)