14. $f(x,y) = 3x^2 - y^2$

a) $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{3x^2 + 6x\Delta x + 3(\Delta x)^2 - y^2 - (3x^2 - y^2)}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{3x^2 + 6x\Delta x + 3(\Delta x)^2 - y^2 - 3x^2 + y^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{6x\Delta x + 3(\Delta x)^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{\Delta x[6x + 3\Delta x]}{\Delta x}$

$f(x+\Delta x, y)$ (x, y)

$= 3(x+\Delta x)^2 - y^2$

$= 3(x+\Delta x)(x+\Delta x) - y^2$

$= 3(x^2 + x\Delta x + x\Delta x + (\Delta x)^2) - y^2$

$= 3(x^2 + 2x\Delta x + (\Delta x)^2) - y^2$

$= 3x^2 + 6x\Delta x + 3(\Delta x)^2 - y^2$

$\lim_{\Delta x \to 0} 6x + 3\Delta x$

$= 6x + 3(0)$

$= \boxed{6x}$

b) $\lim_{\Delta y \to 0} \dfrac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$

$= \lim_{\Delta y \to 0} \dfrac{3x^2 - y^2 - 2y\Delta y - (\Delta x)^2 - (3x^2 - y^2)}{\Delta y}$

$= \lim_{\Delta y \to 0} \dfrac{3x^2 - y^2 - 2y\Delta y - (\Delta y)^2 - 3x^2 + y^2}{\Delta y}$

$= \lim_{\Delta y \to 0} \dfrac{-2y\Delta y - (\Delta y)^2}{\Delta y}$

$= \lim_{\Delta y \to 0} \dfrac{\Delta y[-2y - \Delta y]}{\Delta y}$

$= \lim_{\Delta y \to 0} -2y - \Delta y$

$= -2y - 0$

$= \boxed{-2y}$

$f(x, y+\Delta y)$ (x, y)

$= 3x^2 - (y+\Delta y)^2$

$= 3x^2 - (y+\Delta y)(y+\Delta y)$

$= 3x^2 - (y^2 + y\Delta y + y\Delta y + (\Delta y)^2)$

$= 3x^2 - (y^2 + 2y\Delta y + (\Delta y)^2)$

$= 3x^2 - y^2 - 2y\Delta y - (\Delta y)^2$