# Limits and Continuity

## Definition: Limit of a Function of Two Variables

Let f be a function of two variables defined, except possibly at ($x_o$, $y_o$), on an open disk centered at ($x_o$, $y_o$), and let L be a real number. Then:

$$\lim_{(x,y)\to(x_o,y_o)} f(x,y) = L$$

If for each ε > 0 there corresponds a δ > 0 such that:

$$|f(x,y)-L| < \varepsilon \text{ whenever } 0 < \sqrt{(x-x_o)^2+(y-y_o)^2} < \delta$$

1. Use the definition of the limit of a function of two variables to verify the limit
(Similar to p.904 #1-4)

$$\lim_{(x,y)\to(3,0)} x = 3$$

2. Find the indicated limit by using the given limits
(Similar to p.904 #5-8)

$$\begin{aligned} &Given: \lim_{(x,y)\to(a,b)} f(x,y) = 8, \\ &\lim_{(x,y)\to(a,b)} g(x,y) = 2 \\ &Find: \lim_{(x,y)\to(a,b)} [3f(x,y)g(x,y)] \end{aligned}$$

3. Find the limit and discuss the continuity of the function
(Similar to p.904 #9-22)

$$\lim_{(x,y)\to(4,1)} (x^2 - 2xy)$$

4. Find the limit and discuss the continuity of the function
(Similar to p.904 #9-22)

$$\lim_{(x,y)\to(5,2)} \left(\frac{x^2}{y-1}\right)$$

5. Find the limit and discuss the continuity of the function
(Similar to p.904 #9-22)

$$\lim_{(x,y)\to\left(\frac{1}{4},\pi\right)}\left(x^2\sin(xy)\right)$$

6. Find the limit and discuss the continuity of the function
(Similar to p.904 #9-22)

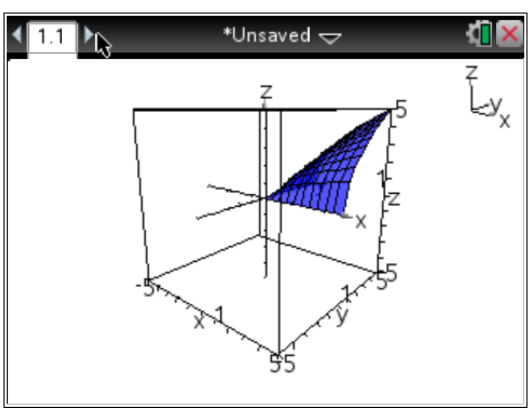
$$\lim_{(x,y)\to(2,2)}\left(\arcsin(x-y)\right)$$

7. Find the limit (if it exists). If the limit does not exist, explain why
(Similar to p.905 #23-36)

$$\lim_{(x,y)\to(0,0)}\left(\frac{x^2-9y^2}{x-3y}\right)$$

8. Find the limit (if it exists). If the limit does not exist, explain why
(Similar to p.905 #23-36)

$$\lim_{(x,y)\to(0,0)}\left(\sqrt{x}\sqrt{y}\right)$$




9. Find the limit (if it exists). If the limit does not exist, explain why (Hint: think along y = 0)
(Similar to p.905 #23-36)

$$\lim_{(x,y)\to(0,0)}\left(\frac{1+y}{x+y}\right)$$

10. Discuss the continuity of the functions f and g. Explain any differences
(Similar to p.906 #43-46)

$$f(x,y)=\begin{cases}\dfrac{9x^4-y^4}{3x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

$$g(x,y)=\begin{cases}\dfrac{9x^4-y^4}{3x^2+y^2}, & (x,y)\neq(0,0)\\ 2, & (x,y)=(0,0)\end{cases}$$

11. Use polar coordinates to find the limit
[Hint: Let x = r cos(θ) and y = r sin(θ), and note that (x, y) -> (0, 0) implies r -> 0]
(Similar to p.906 #53-58)

$$\lim_{(x,y)\to(0,0)}\left(\frac{3x^2y}{x^2+y^2}\right)$$

12. Use polar coordinates and L'Hopital's Rule to find the limit
(Similar to p.906 #59-62)

$$\lim_{(x,y)\to(0,0)}\left(\frac{6\sin(x^2+y^2)}{x^2+y^2}\right)$$

13. Discuss the continuity of the function
(Similar to p.906 #63-68)

$$f(x,y,z)=\frac{z^3}{x^2-y-5}$$

14. Find each limit
(Similar to p.906 #73-78)

$$f(x,y)=3x^2-y^2$$

$$a)\ \lim_{\Delta x\to 0}\frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$$

$$b)\ \lim_{\Delta y\to 0}\frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}$$