8. (cont.)

1ST SEGMENT

$\vec{r}(t) = \underbrace{3t}_{x(t)=3t}\vec{i} \qquad 0 \le t \le 1$

$\int_C (x + 2\sqrt{y})\, ds$

$f(x,y) = x + 2\sqrt{y}$

$f(x(t), y(t)) = 3t + 2\sqrt{0}$

$= 3t$

$x'(t) = 3 \qquad y'(t) = 0$

$= \int_0^1 3t\sqrt{3^2 + 0^2}\, dt$

$= 9\left[\frac{1}{2}t^2\right]_0^1$

$= \frac{9}{2}\left[1^2 - 0^2\right]$

$= \frac{9}{2}$

2ND SEGMENT

$\vec{r}(t) = \underbrace{(-3t+6)}_{x(t)}\vec{i} + \underbrace{(2t-2)}_{y(t)}\vec{j} \qquad 1 \le t \le 2$

$\int_C (x + 2\sqrt{y})\, ds$

$f(x,y) = x + 2\sqrt{y}$

$f(x(t), y(t)) = -3t + 6 + 2\sqrt{2t-2}$

$x'(t) = -3$

$y'(t) = 2$

$= \int_1^2 (-3t + 6 + 2\sqrt{2t-2})\sqrt{(-3)^2 + 2^2}\, dt$

$= \sqrt{13}\left[\int_1^2 (-3t+6)\, dt + \int_1^2 2(2t-2)^{\frac{1}{2}}\, dt\right]$

$u = 2t-2 \qquad du = 2\, dt$

$= \sqrt{13}\left(\left[-\frac{3}{2}t^2 + 6t\right]_1^2 + \int_{t=1}^{t=2} u^{\frac{1}{2}}\, du\right)$

$= \sqrt{13}\left(-6 + 12 - \left(-\frac{3}{2} + 6\right) + \left[\frac{2}{3}u^{\frac{3}{2}}\right]_{t=1}^{t=2}\right)$

$= \sqrt{13}\left(\frac{3}{2} + \frac{2}{3}\left[2t-2\right]_{t=1}^{t=2}\right)$

$\sqrt{13}\left(\frac{3}{2} + \frac{2}{3}\left[2 - 0\right]\right)$

$\sqrt{13}\left(\frac{3}{2} + \frac{4}{3}\right)$

$\sqrt{13}\left(\frac{9}{6} + \frac{8}{6}\right)$

$\frac{17\sqrt{13}}{6}$