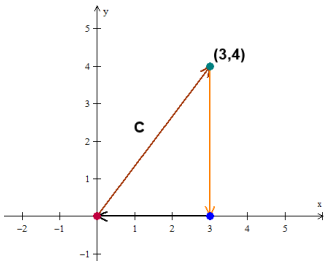
Line Integrals

1. Find a piecewise smooth parametrization of the path C (Note that there is more than one correct answer)
(Similar to p.1079 #1-6)



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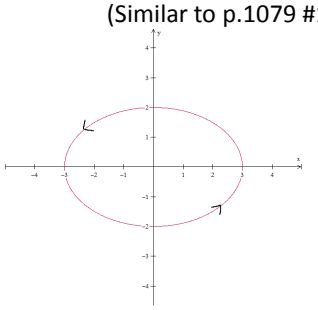

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

3. Find a piecewise smooth parametrization of the path C (Note that there is more than one correct answer)
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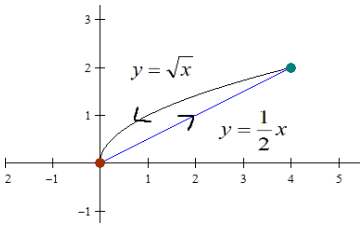


Evaluation of a Line Integral as a Definite Integral

Let f be continuous in a region containing a smooth curve C. If C is given by **r**(t) = x(t)**i** + y(t)**j**, where a ≤ t ≤ b then

$$\int_C f(x,y)\,ds=\int_a^b f\big(x(t),y(t)\big)\sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$$

If C is given by **r**(t) = x(t)**i** + y(t)**j** + z(t)**k**, where a ≤ t ≤ b then

$$\int_C f(x,y,z)\,ds=\int_a^b f\big(x(t),y(t),z(t)\big)\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt$$

4. Evaluate the line integral along the given path
(Similar to p.1079 #7-10)

$$\int_C 2xy\,ds$$

$$C: r(t)=2ti+4tj$$

$$0\le t\le 1$$

5. Evaluate the line integral along the given path
(Similar to p.1079 #7-10)

$$\int_C (x^2 + y^2 - z)\, ds$$

$$C : r(t) = (\sin t)i + (\cos t)\, j + 3k$$

$$0 \le t \le \frac{\pi}{2}$$

6. Find a parametrization of the path C and evaluate the integral
(Similar to p.1079 #11-14)

$$\int_C (3x^2 - y^2)\, ds$$

C : line segment from $(0, 0)$ to $(3, 1)$

7. Find a parametrization of the path C and evaluate the integral
(Similar to p.1079 #11-14)

$$\int_C (x^2 + y^2)\, ds$$

C : counterclockwise around the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(-2, 0)$

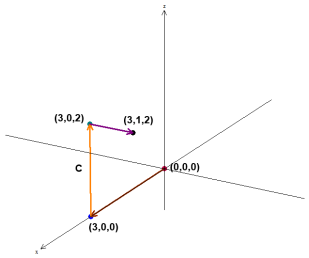
8. Find a parametrization of the path C and evaluate the integral
(Similar to p.1079 #15-18)

$$\int_C (x + 2\sqrt{y})\, ds$$

C : counterclockwise around triangle with vertices $(0, 0)$, $(3, 0)$, and $(0,2)$

9. Find a piecewise smooth parametrization of the path C and evaluate the integral
(Similar to p.1079 #19-20)

$$\int (x + y - z^2)\, ds$$



10. Find the total mass of the wire with density ρ
(Similar to p.1079 #23-26)

$$r(t) = t^2 i + 3tj,\ \rho(x, y) = 2y,\ 0 \le t \le 1$$

$$\text{Hint : mass} = \int_a^b \rho\big(x(t), y(t)\big)\big\|r'(t)\big\|\, dt$$

11. Evaluate the following integral where C is represented by r(t)
(Similar to p.1080 #27-32)

$\int_C \mathrm{F}\cdot \mathrm{dr}$

where

$F(x,y) = x^2 yi + yj$

$C : r(t) = (2\cos t)i + (2\sin t)\,j, 0 \le \mathrm{t} \le \frac{\pi}{2}$

$\text{Hint} : \int_C \mathrm{F}\cdot \mathrm{dr} = \int_a^b F\big(x(t), y(t), z(t)\big)\cdot r'(t)\,\mathrm{dt}$

12. Find the work done by for the force field F on a particle moving along the given path
(Similar to p.1080 #35-40)

$F(x,y) = x^2 i + 3yj$

$C : x = t, y = t^2$ from $(0,0)$ to $(3,9)$

$\text{Hint} : W = \int_a^b F\big(x(t), y(t), z(t)\big)\cdot r'(t)\,\mathrm{dt}$

13. Evaluate the line integral along the path C given by x = 3t, y = 5t, where 0 < t < 1
(Similar to p.1081 #51-54)

$\int_C (2x + 5y^2)dy$

14. Evaluate the integral along the path C
(Similar to p.1081 #55-62)

$\int_C (3x + y)dx + (x + 2y)dy$

$C : \mathrm{x}\text{-axis from } \mathrm{x} = 0 \text{ to } \mathrm{x} = 3$

15. Find the area of the lateral surface over the curve C in the xy-plane and under the surface z = f(x, y)
(Similar to p.1081 #63-70)

$f(x,y) = 3x^2 y$

$C : \text{line from } (0,0) \text{ to } (2,5)$

Hint :

$\text{lateral surface area} = \int_a^b \mathrm{f}\big(\mathrm{x(t)}, \mathrm{y(t)}\big)\big\|\mathrm{r'(t)}\big\|\,dt$