

$$11. \vec{r}(u,v) = (9 \sin u \cos v) \vec{i} + (9 \sin u \sin v) \vec{j} + (9 \cos u) \vec{k}$$

$0 \leq u \leq \pi \quad 0 \leq v \leq 2\pi$

$$\vec{r}_u = (9 \cos u \cos v) \vec{i} + (9 \cos u \sin v) \vec{j} + (-9 \sin u) \vec{k}$$

$$\vec{r}_v = (-9 \sin u \sin v) \vec{i} + (9 \sin u \cos v) \vec{j} + 0 \vec{k}$$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 \cos u \cos v & 9 \cos u \sin v & -9 \sin u \\ -9 \sin u \sin v & 9 \sin u \cos v & 0 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 9 \cos u \sin v & -9 \sin u \\ 9 \sin u \cos v & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 9 \cos u \cos v & -9 \sin u \\ -9 \sin u \sin v & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 9 \cos u \cos v & 9 \cos u \sin v \\ -9 \sin u \sin v & 9 \sin u \cos v \end{vmatrix}$$

$$= 81 \sin^2 u \cos v \vec{i} + 81 \sin^2 u \sin v \vec{j} + (81 \sin u \cos u \cos^2 v + 81 \sin u \cos u \sin^2 v) \vec{k}$$

$$= 81 \sin^2 u \cos v \vec{i} + 81 \sin^2 u \sin v \vec{j} + [81 \sin u \cos u (\cos^2 v + \sin^2 v)] \vec{k}$$

$$= 81 \sin^2 u \cos v \vec{i} + 81 \sin^2 u \sin v \vec{j} + 81 \sin u \cos u \vec{k}$$

so

$$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{(81 \sin^2 u \cos v)^2 + (81 \sin^2 u \sin v)^2 + (81 \sin u \cos u)^2}$$

$$= \sqrt{81^2 [\sin^4 u \cos^2 v + \sin^4 u \sin^2 v + \sin^2 u \cos^2 u]}$$

$$= 81 \sqrt{\sin^4 u (\cos^2 v + \sin^2 v) + \sin^2 u \cos^2 u}$$

$$= 81 \sqrt{\sin^4 u + \sin^2 u \cos^2 u}$$

$$= 81 \sqrt{\sin^2 u (\sin^2 u + \cos^2 u)}$$

$$= 81 \sqrt{\sin^2 u}$$

$$= 81 \sin u$$