Parametric Surfaces

Definition of Parametric Surface

Let x, y, and z be functions of u and v that are continuous on a domain D in the uv-plane. The set of points (x, y, z) given by

r(u, v) = x(u, v)**i** + y(u, v)**j** + z(u, v)**k**

Is called a parametric surface. The equations

x = x(u, v), y = y(u, v), and z = z(u, v)

Are the parametric equations for the surface

1. Graph the vector-valued function
(Similar to p.1109 #1-6)

$$r(u,v) = (u^2 \cos v)i + (u^2 \sin v)\,j + (2u)k$$

2. Find the rectangular equation for the surface by eliminating the parameters from the vector-valued function. Identify the surface and sketch its graph
(Similar to p.1109 #7-10)

$$r(u,v) = (u)i + (v)\,j + \left(\frac{v}{6}\right)k$$

3. Find the rectangular equation for the surface by eliminating the parameters from the vector-valued function.
(Similar to p.1109 #7-10)

$$r(u,v) = (9\cos v \cos u)i + (9\cos v \sin u)\,j + (6\sin v)k$$

4. Find a vector-valued function whose graph is the indicated surface.
(Similar to p.1109 #21-30)

$$\text{The Plane}: \mathrm{x} + \mathrm{y} + \mathrm{z} = 3$$

5. Find a vector-valued function whose graph is the indicated surface.
(Similar to p.1109 #21-30)

$$\text{The Cone}: x = \sqrt{9y^2 + z^2}$$

6. Find a vector-valued function whose graph is the indicated surface.
(Similar to p.1109 #21-30)

$$\text{The Cylinder}: x^2 + y^2 = 36$$

7. Write a set of parametric equations for the surface of revolution obtained by revolving the graph of the function about the given axis.
(Similar to p.1110 #31-34)

$$y = \frac{x}{7}, 0 \le x \le 21 \quad x-axis$$

8. Write a set of parametric equations for the surface of revolution obtained by revolving the graph of the function about the given axis.
(Similar to p.1110 #31-34)

$$y = x^{8/7}, 0 \le x \le 5 \quad x-axis$$

Finding a Tangent Plane to a Parametric Surface

1. Find what u and v are equal to given the point (x_o, y_o, z_o)
2. Find the partials r_u and r_v
3. Find the cross product $r_u \times r_v$
4. Plug in your u and v values into the cross product which gives the normal vector ai + bj + ck
5. Tangent plane is:
   $a(x - x_o) + b(y - y_o) + c(z - z_o) = 0$

9. Find an equation of the tangent plane to the surface represented by the vector-valued function at the given point.
(Similar to p.1110 #35-38)

$$r(u,v) = (9u+v)i + (u-v)j + vk, \quad (3,-3,3)$$

Area of a Parametric Surface

Let S be a smooth parametric surface

r(u, v) = x(u, v)i + y(u, v)j + z(u, v)k

Defined over an open region D in the uv-plane. If each point on the surface S corresponds to exactly one point in the domain D, then the surface area of S is given by

$$\text{Surface area} = \iint_S dS = \iint_D \|r_u \times r_v\| dA$$

$$where\ r_u = \frac{\partial x}{\partial u} i + \frac{\partial y}{\partial u} j + \frac{\partial z}{\partial u} k \text{ and } r_v = \frac{\partial x}{\partial v} i + \frac{\partial y}{\partial v} j + \frac{\partial z}{\partial v} k$$

10. Find the area of the surface over the given region.
(Similar to p.1110 #39-46)

$$r(u,v) = (6u)i - vj + vk, \quad 0 \le u \le 2, \quad 0 \le v \le 4$$

11. Find the area of the surface over the given region.
(Similar to p.1110 #39-46)

$$r(u,v) = (9\sin u \cos v)i + (9\sin u \sin v)j + (9\cos u)k$$
$$0 \le u \le \pi, \quad 0 \le v \le 2\pi$$